ENTRANCE EFFECT AND FILM THICKNESS IN A VERTICALLY FALLING FILM

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NOMENCLATURE

AT	cm.	actual thickness.
C_1	/sec.	integration constant.
C_2	cm./sec.	integration constant.
F	dimensionless	(slip)/(no slip) thickness ratio.
g	cm./sec.^{2}	gravity.
NRE	dimensionless	Reynolds number.
P	gm./cm.-sec.^{2}	pressure.
Q	cm.^{2}/sec.	peripheral flow rate.
RAT	dimensionless	reduced actual thickness.
t	sec.	time.
TNT	cm.	theoretical Nusselt thickness.
u	cm./sec.	velocity in the y-direction.
v	cm./sec.	velocity in the x-direction.
$\bar{v}$	cm./sec.	average velocity.
v_{wall}	cm./sec.	velocity at the wall.
v_{max}	cm./sec.	maximum velocity.
w	cm./sec.	velocity in the z-direction.
x	cm.	coordinate in the direction of flow.
y	cm.	coordinate from the film surface across the thickness of the film.
z	cm.	coordinate across the flow surface.
β	gm./cm.^{2}-sec.	slip coefficient.
δ	cm.	actual thickness with no slip.

Δ	cm.	actual thickness with slip at the wall.
ρ	gm./cm.^{3}	density.
μ	gm./cm.-sec.	viscosity.
ν	cm.^{2}/sec.	kinematic viscosity.
Ω	cm.^{2}/sec.^{2}	force potential.
∇^2	/cm.^{2}	Laplacian operator.

Subscripts

max	maximum value assumed by quantity.
s	value of quantity when there is slip at the wall.
wall	value of quantity at the wall or solid-liquid interface.

SUMMARY

The flow of liquids in thin films has many important applications in chemical engineering. The particular case of vertical flow under the influence of gravity of a falling liquid film bounded by two different surfaces -- the first a solid-liquid interface and the second a liquid-gas interface -- was the subject of this investigation. Although many investigators have contributed a wealth of experimental data covering different liquids and flow configurations, there exist significant disagreements among the various authors in many areas of film flow.

In this investigation the thickness of water films flowing down a vertical plate was measured over a range of flow rates. The effect of viscosity was determined by using a water solution of glycerine, and the effect of lowering the surface tension of water was obtained by adding Aerosol-OT. Measurements of this nature have been previously reported; however, an analysis of the experimental methods employed and the measuring systems utilized indicated that to the present no entirely satisfactory procedure had been devised.

When a liquid is introduced onto the top edge of a vertical flat plate, there exists for a certain distance below a zone over which the liquid flow appears to be smooth and no ripples can be observed. At a certain distance below the point of liquid introduction, this distance being a function of flow rate for a given liquid, there appears on the surface a wave or ripple front. This wave front extends across the entire flow area of the flat plate and is at a constant distance from

the top edge for a given flow rate. Below this wave front, at lengths greater than the distance before wave inception, the liquid surface is in constant motion normal to the direction of the bulk liquid flow.

A single measurement of liquid film thickness is inadequate to characterize the conditions of liquid flow on a flat vertical plate. There are two distinct regions of flow; at the top of the plate the liquid surface is relatively stationary, and at the bottom of the plate the liquid surface is in motion in and out from the surface of the plate and the film thickness is constantly changing. Where the film surface is relatively stationary and the film thickness does not change significantly with respect to time, one thickness measurement might be adequate. Where the film surface is rapidly changing with respect to time, however, a series of film thickness measurements must be made to determine adequately the variation in film thickness. Finally, as has been shown in this work, even at the top of the plate where the film thickness does not vary significantly with respect to time, there is a variation in film thickness with the position on the plate at which the film thickness measurement is made.

A careful review of the literature indicated that most previous investigators had taken film thickness measurements at only one position on the flow area. Others had used methods which would give an average film thickness over the entire flow area or used methods which tended to give a time-averaged film thickness. Because of the previously mentioned characteristics of film flow and the inadequacy of the previous methods of film thickness measuring systems, a new and different film thickness measuring system was developed. An optical system was designed

and built which permitted precise film measurements to be made at any point in the flowing film almost instantaneously, i.e., within 1/1000 second.

The film thickness of three liquids was measured: water, a water-glycerine solution, and an Aerosol-OT-water solution. Film thicknesses were determined at positions on an aluminum plate varying from the top edge of the flat plate to a position about ten inches below the top edge, a position well within the rippled flow region. At each position for a given liquid the flow rate was varied and the thickness at the center line of the flow determined as a function of flow rate. For each liquid solution 20 thickness measurements were made at each position and the 20 measurements were repeated at each flow rate. Over 1500 film thickness determinations were made in the course of the investigation.

The major conclusions of this work include the following:

1. The film thickness of a given liquid at a given flow rate is a function of the position on the plate at which the film thickness measurement is made. In general, the liquid film assumes a minimum film thickness some distance below the top edge of the plate. Rippling breaks out on the film surface and this causes an apparent increase in the average film thickness after the minimum film thickness is attained.

2. A wave amplitude was determined by subtracting the minimum, measured, film thickness from the maximum, measured, film thickness. The wave amplitudes followed the same general behavior as the film thicknesses. They were relatively small at the top of the plate and increased as the distance from liquid introduction increased.

3. The distance before wave inception was determined by a method

more precise than methods reported in the literature.

4. A 27 percent by weight glycerine-in-water solution which has approximately the same density and surface tension as water but twice the viscosity was found to have a greater film thickness than water films flowing at the same Reynolds number, but the film thickness profile followed the same general trend as water.

5. A 0.1 percent by weight solution of Aerosol-OT in water, which has approximately the same density and viscosity as water but half the surface tension, had increased stability and film thicknesses. The stability of the film was increased to the extent that wave motion was not visible on the four foot long liquid surface.

6. An analysis of the deviations of liquid film flow from the theoretical Nusselt prediction indicates that the deviations are in the direction which would be predicted if it were assumed that the liquid solutions were slipping at the solid-liquid interface.

CHAPTER I

INTRODUCTION

The flow of liquids in thin layers is a common occurrence in the everyday life of the layman as well as the chemical engineer. The flow of water in thin layers may be observed on a window pane during a rainstorm, on an inclined roof or in a gutter. It may be noted in this type of flow that the liquid surface is not always smooth but sometimes is disturbed by small wave motions.

In other technical areas not directly associated with chemical processing, Fulford[1] has mentioned a number of situations where the flow of liquids in thin films is encountered including the following:

1. The design of channels, highway bank angles and water drainage systems.

2. The design of dam spillways, drain pipes and overflow lines.

3. Methods of combating soil erosion and the transport of radioactive materials by rain-water runoff.

4. Film cooling of rocket motors, turbine blades and reactor tubes.

5. Cooling of spacecraft by heat transfer from wavy films of molten material.

Examples of moving liquid films bounded on one side by a stationary solid surface and on the other by a gas or vapor phase are common in the chemical processing industry. One of the first uses of liquids

flowing in thin films in the chemical industry was in 1836 when hydrogen chloride gas was absorbed by water flowing through a packed bed.

An understanding of the mechanics of film flow is important in those cases of heat and mass transfer which involve two-phase flow. Flow in vertical condensers, film reactors and packed towers are a few examples of this flow configuration. In the general field of diffusional processes, including absorption, extraction, humidification and distillation, the flow of thin liquid films is often encountered. Filmwise condensation in condensers and in heat exchangers along with the flow of the liquid phase in packed or wetted-wall columns involves the formation and flow of thin liquid films. In the case of absorption where the gas film is controlling, the flow characteristics of the liquid do not appreciably influence the efficiency of operation for a constant area of contact. However, in the treatment of film condensation or in diffusional processes where the resistance of both the liquid and the gas films is significant or where the liquid film is controlling, the flow characteristics of the liquid film have a definite bearing on the efficiency of the operation.

The more exactly the film thickness, interfacial velocity and average velocity of the film can be evaluated, the more precisely the mass transfer and heat transfer characteristics of the operation can be analyzed. Also, with a better understanding of the film-flow characteristics, the dynamic interfacial contact area can be more precisely evaluated, i.e., the effect of ripples in the liquid film on the interfacial area can be taken into account to provide better design information, even in those cases where the gas film is the controlling film

and the liquid film plays a relatively small role in the overall process.

Origin of the Problem at Georgia Tech

In 1962, Stern,[2] in work done at the Georgia Institute of Technology, completed an investigation on the effect of reduced pressure on the performance of a wetted-wall distillation column operating under conditions of total reflux. A wetted-wall column was chosen rather than a packed column in the expectation that the elimination of the column packing as a variable, i.e., the elimination of the liquid-gas contact area as a variable, would permit a more fundamental examination of the effect of reduced pressure on the distillation process.

The vapor stream in a wetted-wall column is bounded by a liquid film on the interior column wall. Hence, the condition of the liquid surface can be expected to influence the vapor stream in much the same way that wall roughness influences flow in pipes. In the region of turbulent vapor flow, a decrease in operating pressure at constant vapor Reynolds number caused an increase in column efficiency. This result is in agreement with mass-transfer theory which assumes that the resistance to mass transfer is the sum of the resistance of the turbulent core and the resistance of the true laminar layer. However, in the region of transitional vapor flow, a decrease in operating pressure at constant vapor Reynolds number caused a decrease in column efficiency. This was shown to be due to the influence of liquid-film rippling on vapor turbulence. It was also shown that the vapor Reynolds number at which turbulent flow began in the vapor was a function of the degree of

rippling of the liquid reflux film and varied with the operating pressure.

A review of the literature was made to determine, if possible, from existing publications the critical characteristics of falling liquid films so that this information could be applied to the above problem in a continuation of the investigation of the efficiency of a wetted-wall column.

Discrepancies in the Literature

It was almost immediately noticed that the points of disagreement in the literature far exceeded the few instances of accord. In the following section, a few of these instances of agreement and disagreement which are particularly pertinent to this investigation are discussed.

Film Thickness

Many of the early film-thickness measurements, which were obtained for the most part by micrometer or simple drainage methods, were not of a high degree of accuracy. However, in the low-flow regions, the values fell near the theoretical thickness predicted by Nusselt.[3] More recent film thickness measurements obtained by improved experimental techniques indicate that in the laminar-flow region the experimentally measured mean film-thickness is lower than the theoretically predicted film thickness of Nusselt for both vertical and inclined flow.

Portalski's[4] experimental results for several fluids flowing as films on a flat vertical plate indicate that at the lowest flow rates the film thicknesses fall well below the Nusselt thickness and deviated

farther from the theoretical thickness as the flow rate decreased. However, these results are not conclusive as other recent measurements by Feind[5] and by Fulford[6] agree with or tend towards the theoretical Nusselt thickness.

Transition from Laminar to Turbulent Flow in Films

In a recent review, Fulford[1] summarized reports in the literature of a transition Reynolds number at which turbulent film flow begins. The time span of these reports covers the period from 1910 to 1962 and the magnitude of the transition Reynolds number reported by various investigators ranged from a low of 400 to a maximum of 3600. Several authors contend that the transition from laminar to turbulent flow is a gradual one which might explain the wide scatter in the reported experimental values. However, the bulk of the experimental evidence seems to support a transition Reynolds number in the range of 1000 to 1600.

Rippling in Film Flow

There are numerous theoretical calculations of the conditions under which rippling occurs in film flow. Most of the treatments predict a critical Reynolds number above which the film flow is unstable and rippled flow occurs. Experimental critical Reynolds numbers determined by careful visual observation, from photographs or by electronic feelers tend to agree with these theories. Unfortunately, the experimental observations mentioned above do not confirm an interesting aspect of the theories on film stability proposed by Benjamin[7] and by Yih.[8] These authors state that film flow on a vertical surface is always unstable and ripples will be present at all flow rates. The use of more sensitive methods of wave detection in recent studies of the phenomena have

led to smaller values of the critical Reynolds number. This seems, perhaps, to be an indirect confirmation of the theory that rippling occurs to some extent at all flow rates.

The Smooth Entrance Length

In nearly all experimental observations of rippled flow, it has been noted that there exists an initially smooth region immediately below the liquid inlet before waves begin to appear on the liquid surface. In general, the length of the smooth entrance region increases as the liquid flow rate increases, but not in a direct proportion. While many authors have made this statement, two notable exceptions exist. Grimley[9] has observed that the films in the entrance region cease to be wavy above a certain liquid flow rate and Batt[10] has observed that below a liquid Reynolds number of about 75, the length of the smooth entrance region increases as the flow rate is decreased. Hence, disagreement exists and further experimental work will be necessary even in what might appear at first to be a relatively simple and straight forward relationship, the length of the smooth entrance region as a function of film flow rate.

Effect of Surface Active Agents on Wavy-Film Flow

The bulk of the experimental data in the literature would indicate that surface-active additives greatly reduce or eliminate surface ripples on a vertically falling liquid water film. Several investigators in rather detailed studies of this phenomena found that as the concentration of the surface-active additive was increased the waves were rapidly damped out until an optimum concentration was reached. The optimum concentration was the concentration beyond which further

addition of the surface-active agent had little noticeable effect on the reduction of surface disturbances. As the concentration was increased beyond this optimum concentration, some authors report little or no effect on the film flow while others report that with continued additions of the surface-active agent the surface will again become rippled. Hence, even without the contradiction of Brotz[11] and Jackson[12] who report little effect of the addition of surface-active agents on film flow there is some question as to what happens as the concentration of surface-active agent is continually increased.

Due to the wide discrepancies in almost every phase of vertically falling, liquid-film flow, it was decided to study further the mechanics of film flow. The objective of this investigation was limited to measurement of the film thickness of a vertically falling liquid film on a flat plate.

Details of Film Thickness Measurements in the Literature

Many different measurement techniques have been employed in the past to determine the thickness of thin films falling on either a vertical plate or on an inclined surface. It has been shown that when a liquid is introduced onto a flat vertical plate, a certain area of the plate below the liquid inlet remains free of any apparent flow disturbance while below this entrance region a definite change occurs in the liquid flow pattern. Here the liquid surface is no longer relatively stationary but is in constant motion normal to the plane of the flow surface. This normal motion is superimposed on the flow parallel to the flat plate. In the entrance region where the surface of the film

is relatively stationary, the film thickness is taken as the distance from the flat plate to the surface of the film. However, in the region well below the entrance region where the film surface is in rapid motion normal to the flat plate, it is not as easy to assign a film thickness. In this region, measurement could be made of the maximum film thickness or the maximum distance from the film surface to the flat plate. In a like manner, a minimum film thickness could be determined when the film surface is at its point of closest proximity to the flat plate.

To date all reported measurements have determined either an average film thickness based on the entire flow area or an average film thickness at one point on the flow plane. Either type of measurement is inadequate to describe what is happening in the liquid film. There is obviously a significant change in flow characteristics from the liquid inlet point to the region of fully developed rippled flow. If the film thickness measurement is made in the entrance region, one thickness measurement will be sufficient at a given point; but, in the rippled region of flow, where the thickness is time dependent, more than a single measurement is necessary.

Most film thickness measurements have apparently been made at a single point in the rippled region of flow. As it will be shown from this work, film thickness is a function of position on the plate as well as flow rate, temperature and the physical properties of the liquid and of the surface of the plate. Hence, it is inadequate to measure the film thickness at only one given point on the flow plate. The general characteristics of the best methods of film thickness measurement which have been employed to date are reviewed in the following sections.

Measurement Methods Involving Surface Contact

The earliest measurements of film thickness in a vertically falling liquid film were made with a method similar to that used by Kirkbride[13] who measured the film thickness of a liquid flowing over the outer surface of a vertical tube. The measurement device consisted of a micrometer mounted at a predetermined distance from the outer surface of the tube. With a liquid flowing down the outer surface of the tube, the micrometer was screwed in until the micrometer arm made contact with the liquid surface. This reading was recorded and then with the liquid flow stopped the micrometer arm further screwed in until it made contact with the outer surface of the tube. By differencing these two readings, the liquid film thickness could be determined. Note, however, the actual film thickness which was being measured by this technique. It was a maximum film thickness, i.e., the thickness of the highest wave crest which passed between the micrometer arm and the cylinder surface, which was measured and by no means an average film thickness.

Hanratty and Engen[14] used the same method for the measurement of liquid film thicknesses in horizontal, parallel flow of air and water streams and claimed an accuracy of 1/64 to 1/32 inch. In the case of horizontal flow, the liquid films are an order of magnitude greater in thickness than the films in vertical flow so that an error of 1/64 inch could be tolerated, whereas in vertical flow the film itself is often less than 1/64 inch thick.

This method of film thickness determination is at best useful only when a true laminar flow exists and no waves are present. Even under these restricting conditions, the precision of the method is not

very good. The inadequacies of this measurement technique were realized by Kirkbride, as well as others, who suggested several alternatives which were later employed by subsequent investigators.

An early modification of the micrometer screw measurement technique was the replacement of the relatively thick micrometer arm with a fine needle-like feeler probe. By using a fine needle which could be adjusted in towards the flowing liquid film rather than the thick micrometer arm, considerable improvement in the measurement of film thickness was realized. The improvement results from the finer control with which the needle can be manipulated and from the smaller disturbance of the flow patterns. The needle contact method as used in later investigations included the further refinement of electrical detection of the contact of the needle with the liquid surface. With this added refinement the needle contact frequency and the length of time the needle was in contact with the liquid surface could be recorded. As the needle was moved closer towards the liquid surface, the length of time the liquid was in contact with the needle increased. When the needle rested in the undisturbed sublayer, the recorder indicated continuous contact of needle and liquid. By making a series of measurements with the needle at varying distances from the flow surface, a maximum, a minimum and a film thickness distribution profile at a given point on the flow surface would be expected. Brauer[15] used this measurement technique for falling liquid films and McManus[16] for two-phase annular flow in tubes. More recently Hewitt, King and Lovegrove[17] did considerable work in the refinement of the above technique.

Regardless of how carefully the above type of measurement is

conducted and how the technical aspects of it are refined, several serious disadvantages exist even if they are of a less restrictive nature than the earlier micrometer arm method. The natural flow characteristics are disturbed by touching the surface of the liquid film with a probe, or even worse, by introducing a probe, however fine, directly into the flowing liquid. Thus, measurement of a disturbed flow pattern is made and this disturbed flow pattern does not necessarily represent the true flow conditions. A second serious disadvantage of this type of method is due to the surface tension of the flowing liquid. The needle probe, once having made contact with the liquid surface, will tend to remain in contact with the liquid; this will give an erroneous indication of the true film thickness. This distorted film thickness will then lead to a false film thickness profile for the liquid film.

Measurement Methods Involving an Electrical Signal

Dukler and Bergelin[18] developed a capacitance method to measure the average film thickness of a liquid film. This method used the flowing liquid surface as one plate of a capacitor and the face of a micrometer probe as the second plate. The capacitance of the capacitor formed by these two plates is directly proportional to the area of the plates and the dielectric constant of the material between the plates (the air gap between the film surface and the face of the micrometer arm) and inversely proportional to the distance separating the plates (the distance between the liquid film surface and the face of the micrometer arm).

To utilize this method, a calibration curve of air gap against capacitance was made under conditions of no-liquid flow. This was done by varying the distance of the micrometer arm face from the plate and carefully measuring both the capacitance and the distance. To make a film-thickness measurement, the capacitance of the two plates with a liquid flowing between them was first determined. Then the distance between the two plates was carefully measured. With the first capacitance measurement and the calibration curve, the air gap with the liquid film flowing could be determined. This air gap could then be subtracted from the distance between the plates, yielding the liquid film thickness.

An inherent assumption in this method is that the surface of the water acts as a conducting plate. A second is that the water surface is flat. The first assumption is probably quite good; however, the second is invalid. Since at practically all flow rates the film surface will be wavy, this method will detect the root-mean-square air gap. In the case of large waves with high frequency, this air gap can differ considerably from the integrated average. By using a small probe and recording the continuously varying capacitance as a train of waves passes under the probe, the wave height as a function of time can be determined. As the probe used by Dukler and Bergelin was 1/8 inch long in the direction of liquid flow, it is questionable whether the film thickness profile thus obtained is a very realistic one. Additionally, the capacitance detector and recorder will have a built-in internal time constant. This time constant will time-average all capacitance measurements to a certain extent resulting in a time-averaged film thickness determination.

The measurement of film thickness by determining its electrical resistance was the method employed by several investigators. Grimley[19] employed this method for vertical tubes, and van Rossom[20] as well as Hanratty and Hershman[21] for horizontal flow over a flat bed. In the horizontal cases, the thickness of the film was determined by the resistance of the liquid film between two electrodes glued to the bottom of the flow plate. The calibration of the device was obtained by direct measurement with a micrometer of a stagnant film. A calibration curve was constructed for use in the dynamic cases by making a series of these measurements with stagnant films of varying thickness. Van Rossom stated that it made no difference whether the calibration film was stagnant or whether a small amount of overflow was provided. Apparently the overflow in question was not sufficient to cause surface waves to form. Hanratty and Hershman used essentially the same method as van Rossom but added that their measurements were open to errors of the order of magnitude of the surface disturbances. With this method, only an average film thickness was reported as no instantaneous measurements were made. Also, in these cases, the size of the resistance electrodes was much larger than the size of the film thickness being measured.

A third measurement technique which is dependent on the electrical properties of the liquid film to determine the liquid film thickness is the measurement of the liquid conductance. This method was discussed quite thoroughly by Hewitt, King and Lovegrove.[17] In their experimental equipment, conductivity probes 1/8 inch in diameter and 1/2 inch from center to center were mounted flush with the wall of a vertical

cylinder. A calibration curve was made from a simulated horizontal arrangement in which two cylinders were separated a short distance from one another and the gap between them filled with the test liquid and its conductivity determined. Severe difficulties were encountered in measuring a mean conductance between these two probes for actual vertical flow due to the rapidly fluctuating conductivities caused by the rippled surface. This problem was solved by an electrical averaging circuit and required the addition of an electrolyte to the test liquid in order to minimize the internal capacitance of the liquid film.

The instantaneous conductance as measured by the conductivity probes was recorded and this gave some indication of the variable fluid surface fluctuations. When this trace was averaged it came out very close to the average as reported by the electrical averaging circuit. While this method does give some indication of the fluid surface conditions, it is the average conductance over at least a 1/2-inch length of liquid surface which was measured. As one complete wave is about this length, the measurement was made over a length interval which is of the order of magnitude of a wavelength. Additionally, the electrical circuitry will time-average the conductivity detected by the conductivity probes.

Photographic Methods of Measurement

Brauer[15] and more recently Belkin, MacLeod, Monrad and Rothfus[22] have used photographic methods to study the behavior of liquid films flowing vertically under the influence of gravity. Belkin and asso-

ciates used two related photographic techniques; one, the direct illumination of the flow surface, and the other a shadowgraph method which produced a silhouette of the wetted flow surface. In both cases they studied the flow of water down the outer surface of a rod one inch in diameter. The film thickness of the flowing liquid was determined by comparing enlarged photographs of the rod under conditions of flow with a similar photograph of the dry rod. Essentially the same method was applied with much greater care by Brauer. A true orthogonal photograph of the flowing film is not obtained but rather an isometric view of the rod and film. This may partially explain the large spread in film thicknesses reported by photographic techniques.

The cross section of a liquid film in annular flow in a horizontal pipe was qualitatively investigated by high speed--1000 frames per second--motion picture photography by Jacowitz and Brodkey.[23] The method they employed was quite unique but has almost-prohibitive restrictions to its use. If a liquid, which has the same refractive index as the glass tube containing the liquid, is used for the film thickness determination, light rays will not be reflected at the liquid-solid interface. The light will traverse the tube and the liquid film in a straight line and the only visible reflection will be at the air-liquid interface. Turpentine proved to be satisfactory for this particular setup. The interface will become visible if a lens with a suitable focal length is focused on the interface. By use of this method the authors confirmed qualitatively that, in horizontal annular two-phase flow, two types of waves exist at the liquid-air interface: low amplitude symmetrical waves and high-amplitude roll waves.

However, no measurements of film thickness were reported.

Light Absorption Measurement Methods

The mean liquid-film thickness was measured by Greenberg[24] by using a photometric method to give the absorption of a portion of a beam of light passing perpendicularly through the liquid film. The procedure measured film thickness in the vertical flow of a liquid in an annular ring on the inner periphery of a tube. The liquid was separately studied in both co-current and countercurrent flow with air. A small beam of light was passed directly through a glass tube. The liquid film in annular flow around the inner periphery of the tube was interrupted by a small rubber separator, which had a hole in its center,permitting the beam of light to pass uninterrupted through the tube and fall on the liquid film surface at a point diametrically opposite the separator. A portion of this incident beam of light was then absorbed by the liquid film, the fraction absorbed being a function of the instantaneous mean film thickness.

Collins[25] modified this technique by omitting the rubber separator. This removed an objection to Greenberg's work in that there was no physical interference with the film flow but it added further complications to the measurement. The incident beam of light now passed through two layers of the liquid film so this would preclude any analysis of the film thickness profile or surface disturbances due to rippling. This also complicated the internal calibration of the light cells since again two varying surfaces rather than one had been interposed between the light source and the light detector. Charvonia[26] also used a

modification of this method. This modification included the reintroduction of the separator in the flow pattern on one of the two diametrically opposed film surfaces through which the incident light beam passed.

An extensive change of the above equipment by Batt[10] allowed the determination of the surface properties of a vertically falling liquid film on the inner surface of a pipe but was unable to determine the actual film thickness. The apparatus as designed by Batt passed a beam of light through one layer of an annular film and intercepted and measured the light rays in the center annular space with a photocell. The electric output of the photocell was relayed to recording equipment outside the test space. Two separate photocells were used. The first was located at the top of the liquid flow section where the liquid surface was smooth and the second was located below this one where the liquid surface was rippled. Apparently, the upper cell was used as a reference against which the lower cell was compared for the authors state: "As previously explained two light beams were employed, and the point of initiation of waves at a particular Reynolds number was consequently taken at that point at which waves were present on the lower trace but not visible on the upper one." This paper presents interesting values for the distance before wave inception and wave frequency; however, the equipment was not calibrated so that wave or film thicknesses could be determined. It should be pointed out that of these photometric methods only that of Batt was designed for single-phase flow. The others, being primarily interested in two-phase flow, could not permit photocells in the annular region. A procedure similar to the above was reported by Lilleleht and Hanratty[27,28] for the case of

horizontal flow over a flat plate.

Deficiencies, in addition to problems concerning the internal calibration of light absorption techniques, arose due to the fact that often the light beam passed through two layers of the liquid film and that in some cases there was physical impairment of the liquid flow. Also, all of the light absorption methods have one further defect. A highly absorptive dye has to be introduced into the water to increase its absorption since water does not absorb a significant fraction of the incident light beam as it passes through the thin flowing film. It is quite possible that the dye had a hydrodynamic effect similar to that of surface-active agents.

Thickness Determinations by Radioactive Methods

A novel method to determine the film thickness of vertically falling liquid films was developed by Jackson[12] using radioactive tracers. A radioactive material (Yttrium-91) was dissolved in water which was allowed to flow as a thin film down the inner wall of a vertical tube. A Geiger-Mueller tube was positioned at the center of the vertical tube through which the water was flowing to detect the radioactivity in the liquid. The film thickness thus controlled the amount of radioactivity seen by the detector. The film thickness represented by a given activity level was not directly determined but was derived from the ratio of the activity at a high liquid flow rate to the activity at a low liquid flow rate. The film thickness at the low liquid flow rate was theoretically calculated according to the Nusselt relation. At the lower flow rates, the activity of the liquid flow was plotted against

the theoretically calculated thickness and this graph was extrapolated for the higher values of flow.

The shortcoming of this method was that it automatically shifted the experimentally determined film thickness at low flow rates to that predicted by Nusselt and, by extrapolating this curve to high flow rates, all film thicknesses were shifted likewise. In addition, it has been shown by several investigators that Nusselt's values are incorrect even at the extremely low flow rates; hence, the film thicknesses determined over the entire range must be in error. Also, this method of measurement will only give an average film thickness and no analysis of film thickness profile can be made.

Direct Weighing of Liquid Holdup

Kamei, Iijima, Kawamura, Oishi and Itoi[29] have developed a sensitive new method for determining the average film thicknesses in wetted-wall tubes. The device employed was a balancing tower which directly weighed the liquid holdup on the wetted surface. The method worked quite well for single-phase studies but the method lost much of its sensitivity when the authors tried to determine film thickness in counter-current gas-liquid flow. Liquid film thicknesses were found to be smaller than those predicted by the theoretical Nusselt analysis.

While this method worked quite well for determining average film thicknesses over the entire flow area, it was incapable of indicating a film-thickness profile. Neither could it give an indication of the changes in film thickness taking place as the liquid proceeded down the wetted flow surface since the film thicknesses were averaged over the

entire plate. Hence, its application to fluid flow studies of liquid films is somewhat limited.

Drainage, or Holdup, Measurements

The drainage, or holdup, method of determining liquid film thicknesses was used 30 years ago by Fallah, Hunter and Nash,[30] a few years later by Friedman and Miller[31] and in the early 1960's by Portalski.[4] The method is relatively straightforward and can be used to measure film thicknesses on the inner or outer wall of a tube or on a flat plate. In this method, the liquid flow was started and held at a constant flow rate. When the flow rate and other experimental conditions, such as temperature have reached steady state, the water supply to the flow plate was diverted and simultaneously a collection of the liquid held on the wetted area began. From the collection of the total liquid layer held on the wetted area and a measurement of this wetted area, the average film thickness was calculated.

Portalski used a refined drainage technique and a carefully designed large-area wetted-wall flow plate to bring to a high level of precision this method of film thickness measurement. As reported, the necessary requirements for a high degree of accuracy are, first, an automatic device for stopping the feed and simultaneously eliminating the weir head and, second, the collection of the drainage from a large wetted area.

This method, when used under the proper conditions, appears to give a high degree of accuracy and reproducibility of results. Indeed, the work by Portalski gives the most voluminous experimental data

available in the literature. However, the thicknesses as reported by this type of experimental determination, like those of the direct weighing method previously discussed, are, by the nature of the experimental design of the measuring equipment, an average film thickness based on the entire area over which the liquid was flowing. Hence, these thicknesses give neither an indication of changes taking place in the film thickness at a given point nor do they indicate the change in film thickness which takes place as the liquid travels down the plate in the direction of flow.

Analysis of Film-Thickness Measurement Techniques

None of the previously used methods of determining the film thickness of vertically falling thin liquid films are entirely satisfactory. Each method suffers from one or more of the following deficiencies:

1. The film thickness measurement is an average value over the entire flow area, or a large portion of it, and does not distinguish between the thickness in the entrance region and the thickness in the disturbed flow area.

2. The film thickness is determined by interrupting the normal flow patterns and therefore measures a disturbed flow rather than the natural flow as desired.

3. The thickness measurement is made at only one point in the flow area, and, hence, the length that the entering liquid has traveled cannot be studied.

4. Electrical averaging apparatus applied to the output of the thickness-measuring device gives a time-averaged rather than an

instantaneous film thickness determination.

5. Some contaminant, such as a dye or an electrolyte, must be added to the liquid under study to facilitate the measurement of an electrical or an optical property of the liquid.

Consequently, it was thought that a better method of determining film thickness would have to be devised before any precise measurement of film thickness at a given point could be undertaken.

Objectives of This Investigation

The objective of this investigation was to measure the thickness of falling liquid films over a wide range of experimental conditions in a manner which would permit the determination of film-thickness variation as a function of time and the position on the plate at which the measurement was made and also to determine the effect of the viscosity and the surface tension of the liquid on the film thickness. Time and distance dependence have not been satisfactorily investigated in the past. In fact, no reference to the effect of position on film thickness was found in the literature in the course of a rather exhaustive survey.

CHAPTER II

EXPERIMENTAL APPARATUS AND PROCEDURE

Description of the Measurement Technique

The apparatus developed for this project makes possible the accurate measurement of the instantaneous film thickness at a given point on a flat vertical plate covered with a flowing liquid film. The measurement is time independent in that there is no thickness averaging and the entire measurement is made in 1/1000 second. The measurement is at an exact point on the plate with respect to axial distance in the flow direction, referred to as the x-axis, and is relatively insensitive to thickness variations on the flowing film surface in a lateral, horizontal or z-axis, direction. Also, no disturbance, chemical or physical, of the liquid-gas interface is required to complete the measurement.

The flat plate used in this experimental investigation was an aluminum plate two feet wide and four feet long. Liquid was introduced evenly across the middle 12 inches of the plate at the top edge. The thickness measurements were made at the centerline of the liquid film, at a point six inches from either edge of the liquid film, at varying distances from the top edge of the plate. The film thickness measurements which were made were point thickness measurements, and not an average thickness across the plate, due to the optical system which was used. This statement will be amplified in a later section. Due to

the wide liquid flow (12 inches) and the relative insensitivity of the measuring system to changes in thickness across the plate in a horizontal direction, the plate was assumed to be semi-infinite with no edge effects.

This film thickness measurement technique is based on the fact that water will absorb light rays while air, relatively speaking, will not. To make a measurement a beam of light is passed almost parallel to the surface of the flat plate. With no water flowing over the plate the surface of the plate, i.e., the base or portion of the plate over which the water flows, will intercept a certain amount of the incident light and allow the remainder to pass over the plate. If a liquid is then allowed to flow over the plate, less light will pass by the surface of the plate since the liquid-gas interface will now intercept the light beam in the same manner that the solid-gas interface did with no liquid flow. However, since the liquid-gas interface is farther from the surface of the flat flow plate by a distance equal to the film thickness less light will be transmitted across the plate. The key to the whole process is to determine accurately the location of the gas-solid interface when no liquid is flowing and then to determine accurately the gas-liquid interface, which is a variable quantity, when the flowing liquid is introduced onto the flat plate.

The method used to determine the interface location was an unusual optical system developed for this experimental program. Two stationary reference points were required for the measurement; the flat plate provided the first and a needle point, referred to hereafter as the reference pin, set at a fixed distance from the plate, provided the second.

The liquid-gas interface, whenever liquid was flowing, fell between these two stationary reference points. A large aerial camera lens was focused on the vertical plane which included the reference pin and which was normal to the flow plate. The image of the pin and the surface of the flow plate were focused on a plane about 15 feet away. Thus, on this plane, the interface at the gas-solid or the gas-liquid boundary could be measured as a function of its distance from the reference pin.

At this point, 15 feet from the object, the magnification of the interface variations (film thickness) was about ten fold. Rather than measure the rapidly fluctuating interface location directly at this point, the light boundary, which located the interface, was used to expose a photographic film at controlled time intervals. The film was placed in a 35 mm. camera in the usual way, but the lens was removed. The camera thus served only as a film holder and shutter. In this manner a permanent record of the interface location was made. A schematic diagram of the system giving the relative location of the components is given in Figure 1.

In practice, light passing through the negative was projected on the wall by an ordinary filmstrip projector rather than take a given measurement directly off a print made by exposing the photographic film. The enlargement of the 35 mm. film on projection was about 35 fold giving an overall magnification for the measurement system of about 350.

Experimental Equipment

The experimental equipment can be divided into two major parts.

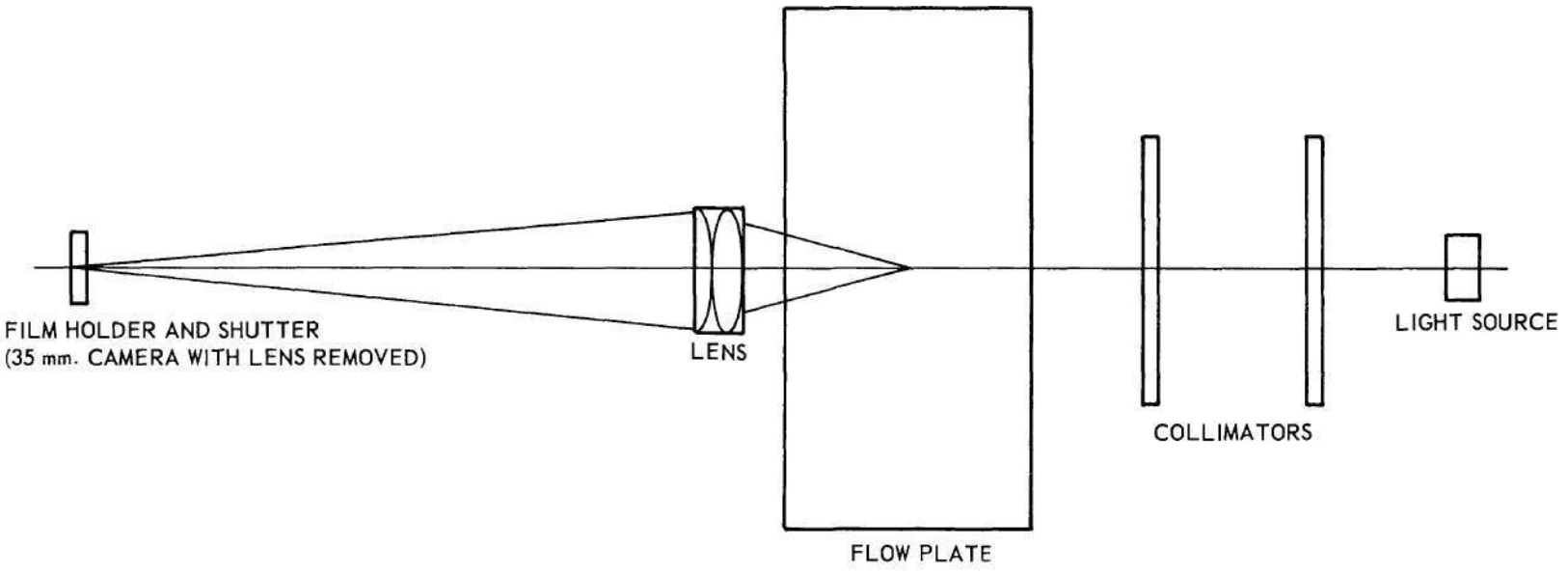

Figure 1. Schematic Diagram of the Measurement System.

The first of these deals solely with the measurement technique while the second deals with the flow properties of the system.

General

The flow system incorporated one unique design feature. Since the entire measuring system was locked into place and held immovable in order to measure the film thickness at varying points over the flat plate flow surface, this flat surface was positioned relative to the measuring system. In the more conventional approach usually taken the position of the measuring system rather than the flow system is adjusted. The flow system consisted of several major components: the upper collection tank which served as a constant head device to insure constant flow, a header mounted above the flow plate containing calming baffles to insure smooth introduction of the liquid onto the flat flow plate, the flat flow plate itself, and a bottom collection tank to collect and contain the run-off from the flat plate.

Figure 2 is a side view of the upper section of the flow system. It shows the upper constant-head tank with its overflow lines and the piping through the rotameter which indicated the liquid flow to the header. Figure 3 shows the lower section of the flow apparatus, including the lower portion of the flat flow plate and the bottom collection tank. Included in this view is the recirculating pump which was used in the glycerine and Aerosol-OT tests to transfer the liquid run-off from the collection tank back to the constant head tank.

As mentioned, the flow surface had to be brought in line with the measuring system rather than the more usual convention of positioning the measuring system. This was accomplished by mounting the flow

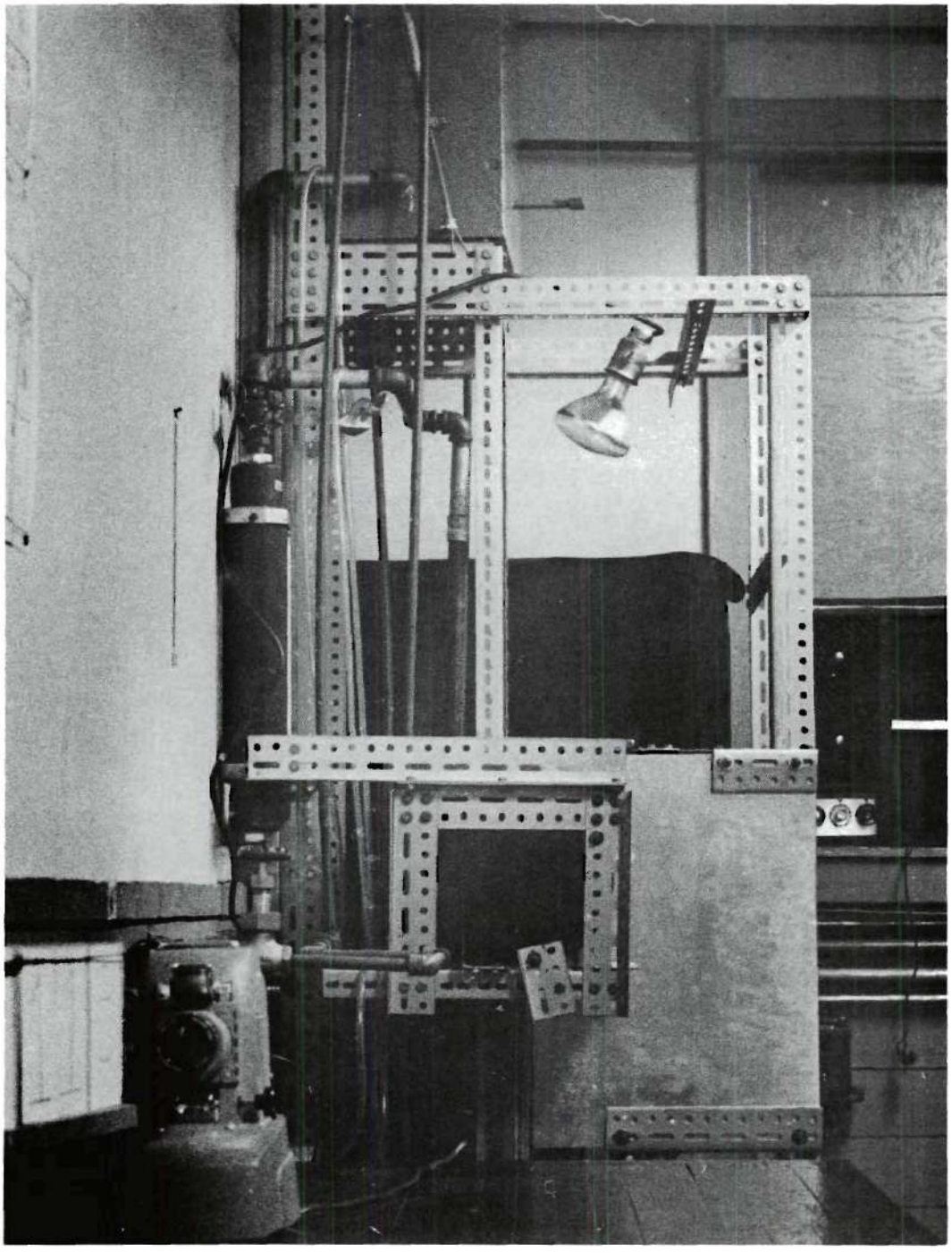

Figure 2. Side View of the Upper Portion of the Experimental Equipment.

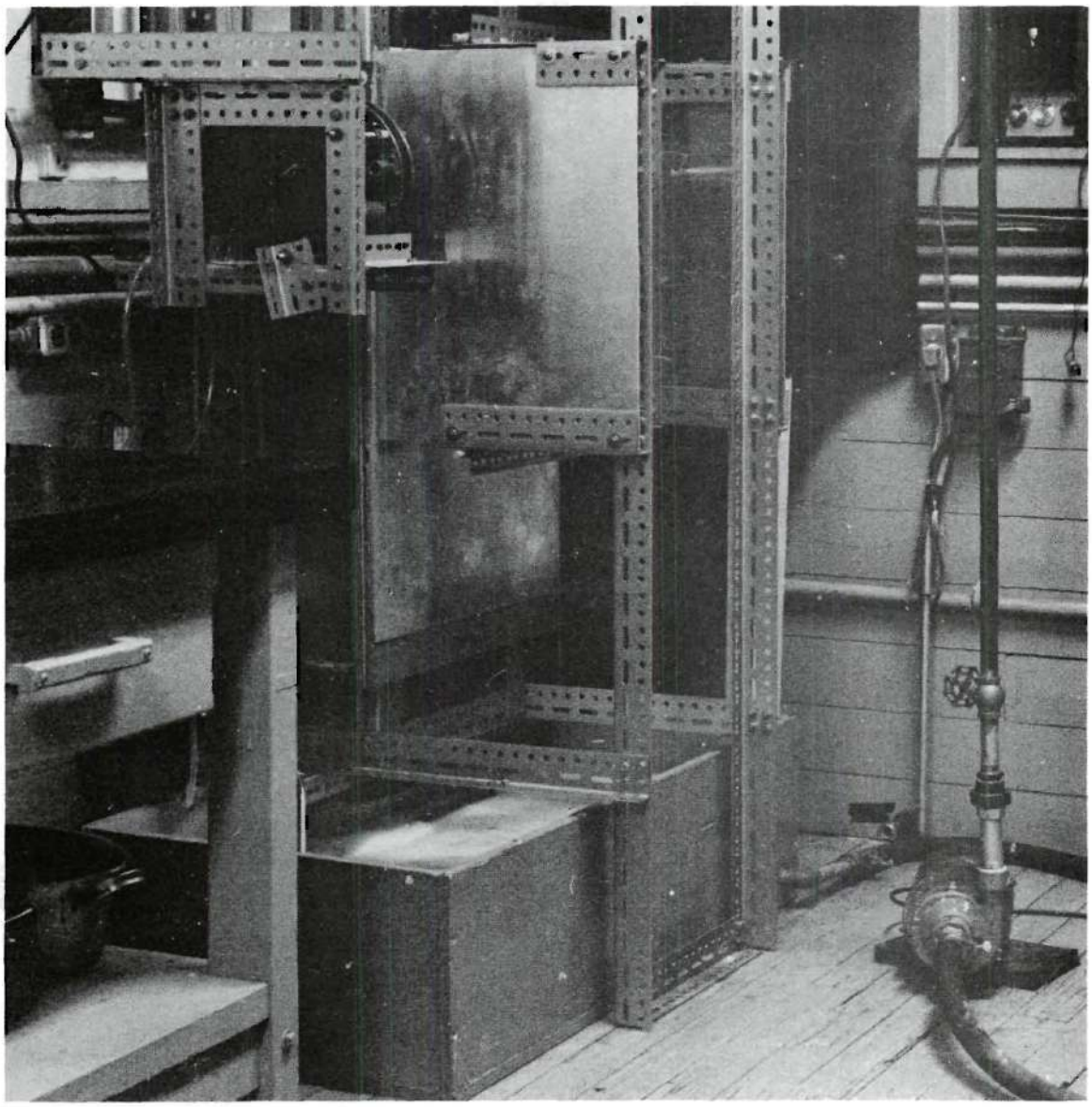

Figure 3. Orthogonal View of the Lower Portion of the Experimental Equipment.

plate and the liquid calming header on a rigid frame. The rigid frame was constructed of angle iron which had been pre-drilled to provide regularly spaced holes along its side. This frame was built so as to fit into the riders of the main equipment--the main equipment being securely fastened to the wall. Thus, this frame could be moved up and down the stationary part of the equipment and placed in any desired location along the axis of the measuring system. Again, except for the flat flow plate and the calming header, the remainder of the flow and measurement equipment was securely fastened to the stationary portion of the experimental frame.

The movable frame and the attached calming header can be seen in Figure 4. Also shown in the position of the lens and pin relative to the flow plate. This figure along with Figure 2 shows the essential details of the measuring system.

Construction Details

The details of construction of the individual components of the measuring system and the flow system are given in Appendix I. Also included in this section of Appendix I are the details of the film processing procedures which were used.

Experimental Procedures

All of the measurements made with a given liquid at a fixed flow rate were designated a series. A series consisted of a number of film thickness measurements at several different points on the plate. Each measurement was made along the center of the plate in a horizontal direction and the points differed from one another in the distance from

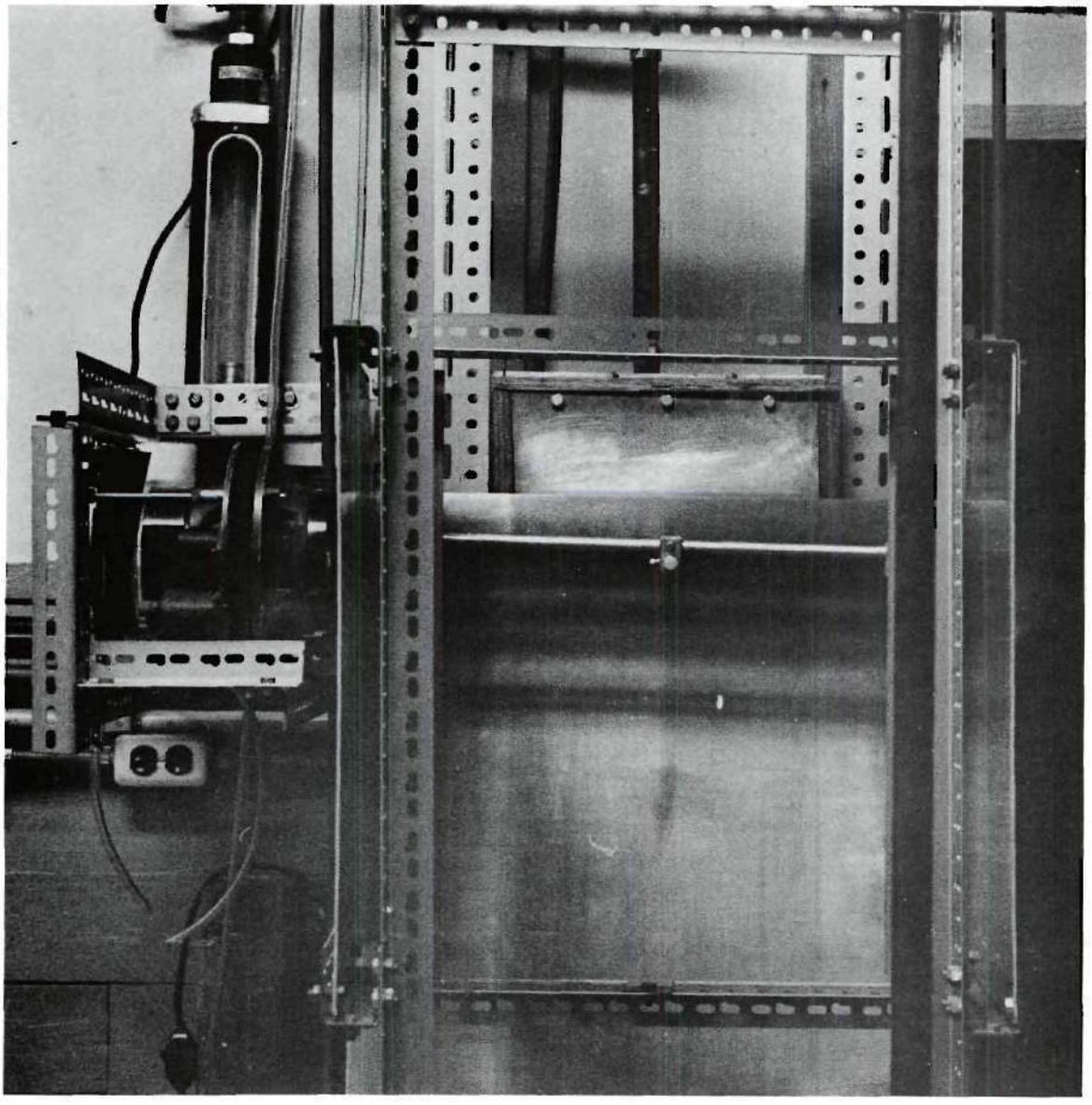

Figure 4. Front View of the Measuring System Details.

the top edge of the plate where the liquid was introduced. In other words, the different positions corresponded to a change in location on the x- or vertical-axis. Each series was unique in that either the flow rate was changed or the liquid under study was changed from that of previous series.

Many instantaneous film thickness measurements were made at each point on the plate. They were called a run when made with a given liquid at a fixed flow rate and at a fixed position on the plate. Hence, each series consisted of a variable number of runs. A change in flow rate or a change in liquid composition distinguished one series from another, whereas, within a given series, a change in position on the plate distinguished one run from another.

A run consisted of 20 measurements of film thickness. Five measurements of the distance from the reference pin to the surface of the flat plate were made in addition to the 20 film thickness measurements in each run. These five measurements were for calibration purposes, as will be explained in a later section. Seventy six runs were made and over 1500 individual thickness measurements were recorded in the course of this study.

The choice of position on the plate at which each run was to be made was not entirely arbitrary due to the method of construction of the experimental equipment. The flat, movable, plate was fastened to the stationary equipment rack using pre-drilled angle iron. As a result, the plate could only be positioned at discrete intervals such that the holes in the riders to which the flat plate was fastened would align with the holes in the stationary equipment rack. Each run was

made in one of these prescribed positions.

Each series, composed of its individual runs, gave the film thickness as a function of distance the liquid had traveled down the plate. A film thickness profile with respect to distance was thus obtained. Each series was started by measuring the film thickness at the top edge of the plate, this being the point of liquid introduction. This position was designated Position 0. After the run at Position 0, the flow was stopped and the plate was moved upward one position to Position 1. Position 1 was located 5/8 inch below the edge of the plate. By moving the plate upward, the stationary measuring system viewed a lower position on the plate. At this position, Position 1, the second run of the series was made. At the completion of the second run in the series the plate was again moved upward until the next set of holes became aligned; this was Position 2. Position 2 was located 1-3/4 inches below the edge of the plate. In a like manner a series of runs was completed by moving the plate upward from one position to the next until the termination of the series.

The number of runs varied from series to series and depended on the length from the top edge of the plate at which rippling was detected. The surface of the liquid film was moving normal to the flow direction in each position at all times but, as will be discussed in a later chapter, near the top of the plate this motion was relatively small whereas near the bottom of the plate the motion was relatively large. There was a point on the plate that was found to be a function of liquid composition and flow rate above which the motion was slight and below which the motion was large. It was this point which is referred to as

the length from the top edge of the plate at which rippling was detected.

After rippling was detected, a few more runs were made in the succeeding positions down the plate and the series was terminated. The position at which the series was terminated varied from Position 5 to Position 9. The relation between position number and distance from the top edge of the plate is given in Table 1.

Film Thickness Measurement

Each experimental run for the determination of liquid film thickness was conducted in the following manner: The flat aluminum flow plate was set at a given position number aligning the intended test position with the light source, reference pin and camera center-line. All bolts holding the test section, or flat plate, to the stationary experimental frame were set in place and tightened. The liquid flow was adjusted and allowed to run at the maximum flow rate for at least 20 minutes. This 20-minute flush cleared the flat plate of any dust which might have settled on the plate since the last run. This preliminary flow period also allowed the temperature of the water and system to equilibrate. This was an important factor because the water line leading to the equipment ran parallel and in close proximity for a considerable distance to the steam line providing heat for the building. The steam line, functioning as a single-pass, co-current heat exchanger sometimes elevated the water temperature as much as 5^{o} C. After a 20-minute flush the water temperature did not vary a detectable amount during the time period required for a run; the water temperature, however, was not the same for each run but was dependent on the ambient

Table 1. Axial Distance Down the Flow Plate in the Direction of Liquid Flow in Terms of Axial Position Number

Axial Position Number	Distance From Top of Flat Plate (inches)
Position 0	0
Position 1	5/8
Position 2	1-3/4
Position 3	2-5/8
Position 4	3-3/4
Position 5	4-5/8
Position 6	5-3/4
Position 7	6-5/8
Position 8	7-3/4
Position 9	8-5/8

conditions at the time of the run.

At the end of the 20 minute preliminary period the flow rate of the liquid was adjusted to that required for the forthcoming run. At this time the reference pin was adjusted to clear completely the surface of the flowing liquid and, in the case of rippling regions, to clear completely the liquid surface even when the waves of largest amplitude passed the reference position on the plate. The liquid flow was then stopped and the camera positioned to include the entire area of interest in the field of view, i.e., the head of the reference pin or the reference point and the surface of the flat plate or base line.

Again the liquid flow rate was adjusted to the proper rate for the run under consideration. At this time an identification number (Run Number) was assigned to the run and entered on the data sheet. The camera was loaded and the first exposure on the roll of film was of the identifying number. The following information was then entered on preprinted data sheet for the run:

1. Date

2. Time

3. Position Number (The axial distance down the plate at which the individual run was being made.)

4. Rotameter Float (Two floats were used in the rotameter monitoring the flow rate of the liquid to the flat plate, an aluminum float for the water and water with Aerosol-OT runs and a stainless steel float for the glycerine runs. The appropriate float was identified.)

5. Liquid (The liquid being used for a given run, water, water with Aerosol-OT or glycerine was entered.)

6. Liquid Temperature

7. Air Temperature

8. Barometric Pressure

Entering the above information on the data sheet completed the preliminary preparation for the run. The same procedure was followed for each film-thickness test. At this point in the procedure a final check was made to spot any random variations in the flow pattern and to see that the flow rate was at the predetermined value.

The cooling fan for the light source was turned on and the light turned on shortly afterwards. The width of flow over the flat plate was measured and recorded on the data sheet. The rotameter reading was recorded. All lights in the test area other than the 1000 watt light source were shut off and 20 exposures were made at 1/1000 second of the flowing liquid film. The exposures were made at random about every five seconds. The lights were turned on and immediately the rotameter reading and width of the film were measured again and recorded on the data sheet. The liquid temperature was checked after the run but in no case was it found to differ from the temperature recorded at the beginning of the run.

The liquid flow was stopped and the flat flow plate dried with a paper towel. Five exposures were then made, again with room lights out and light source on, to establish the distance from the reference pin to the base line. The final exposure on each roll of film for a given run was a repeat of the run number for identification purposes. If another run was made immediately in which only the liquid flow rate was changed the 20-minute flush was not repeated; otherwise, the entire procedure

was repeated for the following run.

The same procedure was followed for each of the three liquids investigated: water, water-glycerine solution, and water-Aerosol-OT solution. The only difference was that during the water runs the water, after passing the test section, was discarded. During the runs for the other two solutions, recirculation of the test solution was employed.

The method for obtaining the film thickness measurement from the exposed photographic film was relatively simple. Prior to the start of the liquid flow an exposure was made of the plate and the reference pin. A side view of the measuring equipment system in the vicinity of the fluid interface is shown in Figure 5. This exposure resulted in a negative simulated by Figure 6. From this negative the distance from the tip of the reference pin to the surface of the flow plate was determined. This reference distance is the distance between lines AA and BB, i.e., AB, in Figure 6. Then a second exposure was made with the liquid flowing over the plate. From this second exposure the distance from the tip of the reference pin to the surface of the flowing liquid was measured as CD, the distance between lines CC and DD in Figure 7. The difference of these two distances, AB-CD, is the relative thickness of the flowing liquid film. This relative distance can be converted to the actual film thickness when the total magnification of the system is known. The total magnification is discussed in the following section. Lines AA on Figure 6 and CC on Figure 7 are identical, as nothing was moved from the time of the first photograph simulated in Figure 6 to that of the second photograph simulated in Figure 7. The base line which was the surface of the flat plate, line BB on Figure 6, is hidden by the

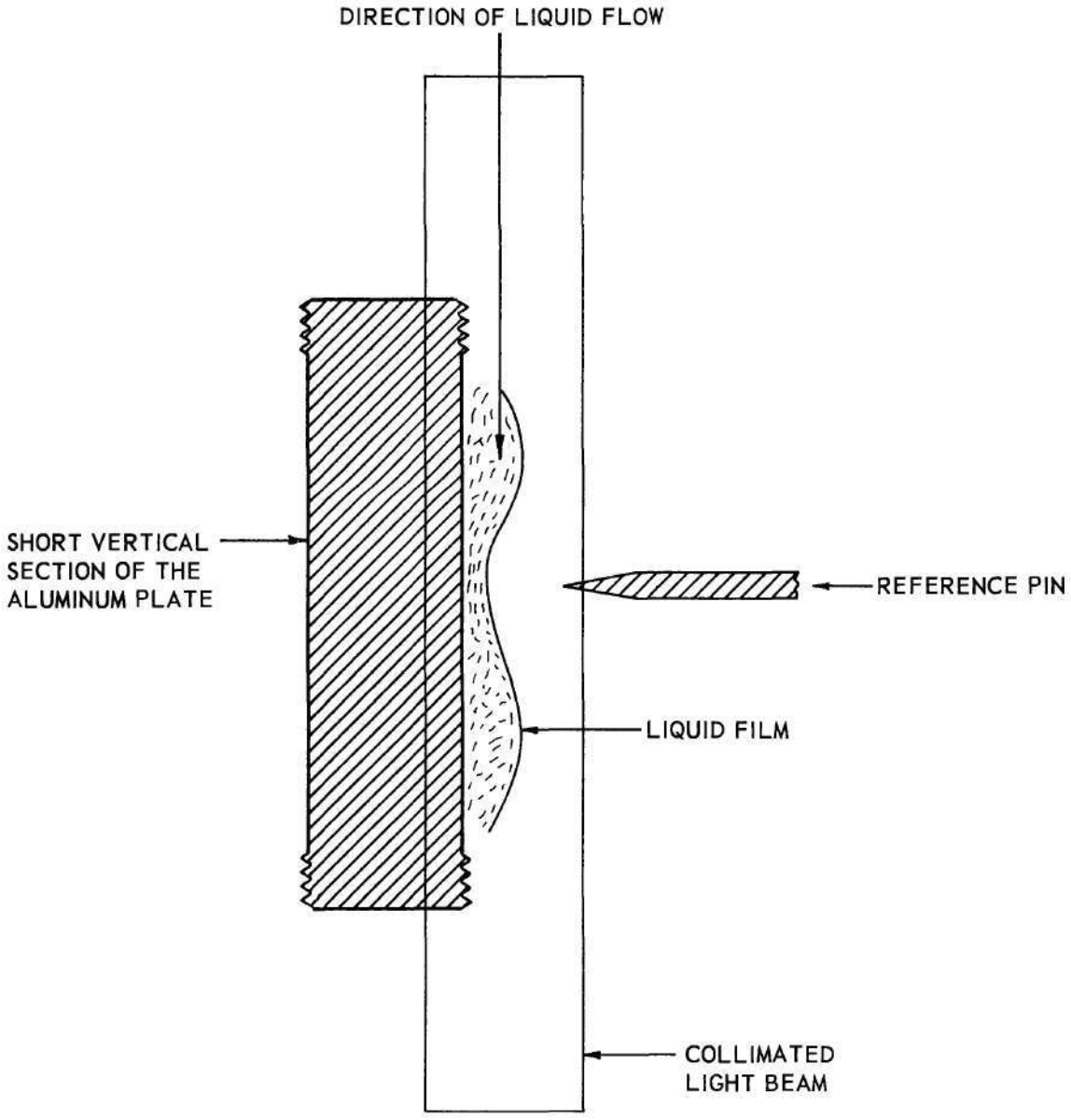

Figure 5. Side View of the Measurement System.

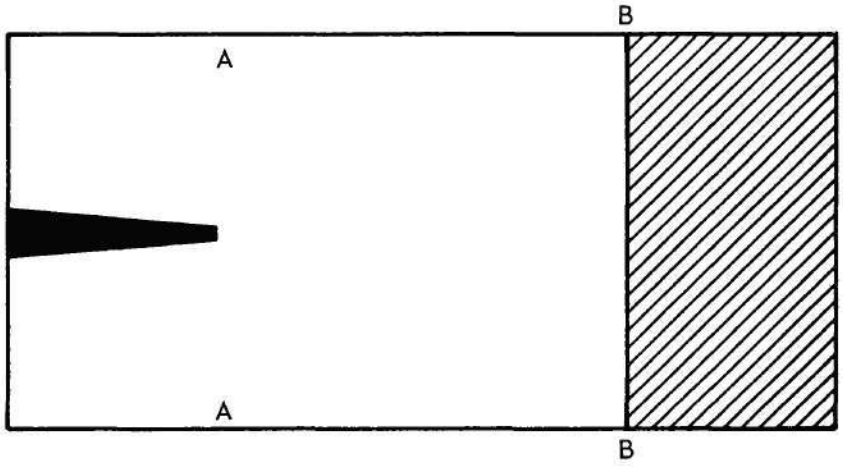

Figure 6. Simulated Photograph of the Base Line.

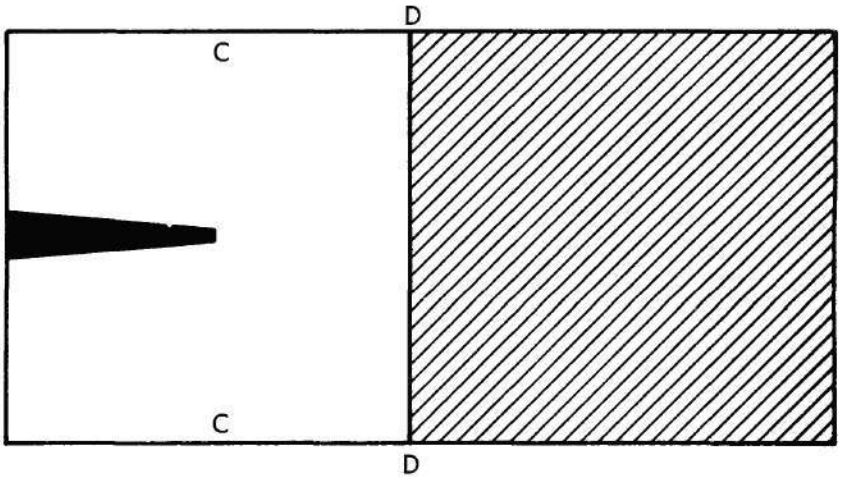

Figure 7. Simulated Photograph of the Liquid Film.

liquid film in Figure 7.

Calibration of Experimental Equipment

The procedure for calibration was much simpler than tests to determine film thickness since there was no liquid flowing during calibration. The general procedure was to set the reference pin at a convenient distance from the flow plate, generally about 30 mils, and assign a reference number to the run. The reference number, as before, was the first exposure on the roll of film corresponding to a given run as well as the last exposure. In this manner it was easy to identify the film as pertaining to a given set of conditions after the film had been developed. The roll for each run contained enough film so that 25 exposures could be made of the experimental conditions plus two for identification.

The first five exposures of a given roll of film, or a given run, were of the flow base line or the surface of the flow plate. This set the distance between the reference pin point and the plate. These exposures were followed by four groups of five exposures, each group under different conditions. In the calibration of the equipment it was necessary to know what effect the individual components had on the magnification of the overall measuring system. To obtain this, measurements were made of a known thickness. This was accomplished by placing gauges of varying thickness between the reference pin and the flat plate. Gauges are strips of metal of known thickness. Generally, in a calibration run, four different sized gauges would be individually placed, one at a time on the flat plate, and measured.

Reproducibility of a Given Thickness Measurement

The film thickness was determined by measuring the magnified image of the actual film thickness on a square grid, 1/10 inch by 1/10 inch, located about 15 feet from the film projector. This apparent thickness was then divided by the magnification of the measuring system, which is discussed in the next section, to obtain the actual film thickness. The apparent film thickness was measured to 1/10 inch. Even for an identical thickness there is an error introduced in film thickness as the second time the thickness was determined it might be 1/10 inch larger or 1/10 inch smaller than the first determination due to the judgment of the operator. This error was more significant in the thinner films than in the thicker films. The percentage error introduced by this reading error for each series is shown in Table 2.

The Overall Multiplication Constant

Two factors contributed to the overall multiplication of the actual film thickness to the much larger image of that thickness which was physically measured after projection on the wall. The first was the magnification due to the 345 mm. focal-length aerial camera lens which was focused on the reference pin point and transmitted the image of the liquid-gas interface to the photographic film. The second factor was the enlargement of the 35 mm. film prior to measuring the apparent image size. The film, after being developed, was projected onto a wall about 12 feet distant from the projector. The measurement of the apparent image size was taken at the wall.

Whatever the mechanism of the magnification process, the important

Table 2. Percent Error in the Film Thickness Measurement Due to the Error in Reading the Apparent Thickness From the Projected Negative.

Series	Percent Error
Water	
Low	3 - 4
Low-Middle	2 - 3
High-Middle	2
High	1 - 2
Glycerine Solution	
Low-Middle	2
High	1
Aerosol-OT Solution	
Low	3 - 4
High-Middle	2
Aerosol-OT Solution on a Waxed Plate	2

question was: if a known thickness was placed between the reference pin and the plate, what final measurement corresponded to that known thickness? The actual thickness of the interposed material and the apparent thickness of this material which was measured on the wall, were related by the overall multiplication constant, M, as follows:

$$(\text{actual thickness}) \times M = (\text{apparent thickness})$$

The constant M could, of course, be determined if each of the two factors mentioned above, the magnification due to the lens and the magnification due to the projection of the developed film, were evaluated. The magnification due to the projection of the developed 35 mm. film onto the wall could be easily determined. However, thick lens optical relations, required to evaluate the magnification due to the lens, necessitate the determination of the several focal lengths involved in the lens. This, in turn, requires very precise physical measurements which would be treated with confidence only if performed by an expert. However, all of the information that was necessary for the calculation of M was at hand when a gauge of known thickness was placed between the reference pin and the flow plate and the usual measurement and processing procedure was followed.

In order to obtain a reliable value for M, the overall multiplication constant, at the start of the experimental phase of this investigation, the value of M was determined for a group of gauges of different known thicknesses varying from 6 mils to 30 mils. In addition, the value of the multiplication factor was checked periodically throughout the experimental phase of the project. Another comprehensive set of

determinations of the absolute calibration of the measuring equipment, including duplicate values for comparison, was made about halfway through the experimental program. The results of the overall magnification constant from both of these determinations are presented in Table 3. Series A is the series run at the start of the experimental phase of this investigation and Series B is the series which was run about halfway through the experimental program.

A valid objection to the above calibration would be that, if the known thicknesses which were interposed between the reference pin and the plate were inaccurate, the film thickness determinations would also be inaccurate. The gauges used were a set of machinist's gauges and were as accurate as could be obtained. However, the calibration was not based on a single gauge. A series of seven gauges was used for calibration, and it can be seen from Table 3 that all gauges gave approximately the same result.

The reproducibility of the different magnification constants varied from the thicker gauges to the thinner gauges. For the 30 mil gauge, the reproducibility was within one percent, whereas for the six mil gauge, the reproducibility of the measurement had closer to a five percent possible error. The reproducibility within each of the two series as well as the reproducibility between the two series would indicate that the overall magnification of the measuring system is very close to the value of 363; this number was used for this work.

A second factor affecting the precision of the calibration was that M is dependent on the location of the 345 mm. lens, the camera, the projector and the viewing screen, in this case the wall. In the

Table 3. Experimental Values for the Overall Multiplication Constant

Probe Thickness Mils	M Series A	M Series B
30	369	363
25	368 (368)	357 (362)
20	377	346
15	363	360
12	358 (362)	357 (342)
9	373	388
6	347 (353)	375 (352)
Average	365	361

course of all experimental tests each of these variables was held constant. The lens was bolted to the stationary framework of the experimental equipment and was immovable. The camera was carefully located, and while it was moved between experimental runs for storage, it was always returned to its marked position prior to a run. In a like manner the projector was moved for storage from time to time but it also was carefully returned again to the same position prior to use.

The individual components due to the lens and the projection of the film could be evaluated after the determination of the overall magnification constant. The long dimension of the 35 mm. film is actually 36 mm., and this length, after being projected on the wall, was measured. This distance was found to be 47.8 inches. As this length corresponded to 36 mm. on the negative prior to projection, the projector magnification was found to be

$$\frac{(47.8 \text{ in.}) \times (25.4 \text{ mm./in.})}{36 \text{ mm.}} = 33.7$$

and since the overall magnification was 363

$$M_1(\text{Projector}) \times M_2(\text{Lens}) = M_o(\text{Overall Magnification})$$

$$33.7 \times M_2(\text{Lens}) = 363$$

which gives

$$M_2(\text{Lens}) = 10.8$$

This value for the lens magnification compares well with the value of 11.2 obtained when the thin lens assumption was made for the aerial

camera lens and the lens magnification calculated. This agreement is well within the limits to be expected for the thin lens approximation applied to a thick lens.

Determination of the Thickness Variation in the Center of the Plate When the Thickness Changed in a Lateral Direction

The determination of the film thickness depended on the amount of light which passed between the reference pin and the moving liquid surface. The effect of film thickness variation in a lateral, or horizontal, direction as measured at the center of the plate was determined. The effect was investigated by first determining the distance from the reference pin to the surface of the plate. After this had been done, a 10 mil-thick gauge was securely fastened to the plate directly in front of the reference pin. This distance was also photographically determined. Without moving the reference pin or the 10 mil gauge, a 12 mil gauge was placed one inch to the right, i.e., between the 10 mil gauge and the light source. The distance between the 10 mil gauge and the reference pin was again photographically determined. This procedure was repeated placing the 12 mil gauge two inches to the right and one and two inches to the left of the 10 mil gauge and each time determining the thickness of the 10 mil gauge by the measurement system.

The same procedure was repeated using a 15 mil gauge in place of the 12 mil gauge and placing it one, two and three inches to the right of the 10 mil gauge and one and two inches to the left. The apparent change in thickness of the 10 mil probe as a function of the size and position of the interfering gauges is given in Table 4 as the percentage change in apparent film thickness. From Table 4 it may be noted that, when the

Table 4. Effect on the Film Thickness at the Center of the Plate of an Increased Thickness at a Lateral Position on the Plate

Position of Thick Probe	Apparent Thickness Change %				
	2"L	1"L	1"R	2"R	3"R
Thickness of Probe					
12 Mil	-0.5	-6.4 (-7.6)	0.9 (2.3)	1.6	
15 Mil	-0.5 (2.0)	3.2	11.0	3.7 (1.0)	2.3

L means to the left of the reference pin which is at the center of the plate.

R means to the right of the reference pin which is at the center of the plate.

interfering gauge was two inches from the center of the plate, the effect on the measured film thickness at the center was usually less than three percent. Since precision of this order was about the limit of this equipment, interfering gauges or surfaces at distances greater than two inches from the center of the plate have essentially no effect on measured film thickness even when the interfering gauge or surface was 50 percent thicker than that at the center. Duplicate values which were taken on separate days from the bulk of the data presented in Table 4 are given in parenthesis in the table.

Flow Rates and Physical Properties of the Liquid Studied

Eight series of experimental runs were made in this study. Each series was unique in that either a different liquid was used or the flow rate of a given liquid was changed considerably from that of the previous series. Water was used at four different flow rates for four of the eight series. The four flow rates for water which were used were:

1. Low Flow Rate — Nominal Reynolds Number of 75
2. Low-Middle Flow Rate — Nominal Reynolds Number of 215
3. High-Middle Flow Rate — Nominal Reynolds Number of 460
4. High Flow Rate — Nominal Reynolds Number of 680

The actual Reynolds numbers for each series which the nominal values above represent are discussed later. The physical properties; viscosity, density and kinematic viscosity, of the test solutions studied are presented in Tables 5 and 6. Table 5 gives the data for the water series; these data were obtained by interpolation from values presented in The Handbook of Chemistry and Physics.[32]

Table 5. Physical Properties of the Water and Aerosol Solutions

Temperature °C	Viscosity gm./cm.-sec.	Density gm./cm.^{3}	Kinematic Viscosity cm.^{2}/sec.
12	1.2363	0.999498	1.2369
12 1/4			1.2286
12 1/2			1.2202
12 3/4			1.2118
13	1.2028	0.999377	1.2035
13 1/4			1.1956
13 1/2			1.1876
13 3/4			1.1797
14	1.1709	0.999244	1.1718
14 1/4			1.1642
14 1/2			1.1566
14 3/4			1.1490
15	1.1404	0.999099	1.1414
15 1/4			1.1341
15 1/2			1.1268
15 3/4			1.1196
16	1.1111	0.998943	1.1123
16 1/4			1.1052
16 1/2			1.0982
16 3/4			1.0912
17	1.0828	0.998774	1.0841
17 1/4			1.0774
17 1/2			1.0708
17 3/4			1.0641
18	1.0559	0.998595	1.0574
18 1/4			1.0509
18 1/2			1.0444
18 3/4			1.0380
19	1.0299	0.998405	1.0315
19 1/4			1.0253
19 1/2			1.0192
19 3/4			1.0130
20	1.0050	0.998203	1.0068
20 1/4			1.0008
20 1/2			0.9949
20 3/4			0.9890
21	0.9810	0.997992	0.9830
21 1/4			0.9772
21 1/2			0.9715
21 3/4			0.9658
22	0.9579	0.997770	0.9600

Table 6. Physical Properties of the Glycerine Solution

Temperature °C	Viscosity gm./cm.-sec.	Density $gm./cm.^3$	Kinematic Viscosity $cm.^2/sec.$
20	2.242	1.06477	2.106
20 1/4			2.091
20 1/2			2.077
20 3/4			2.063
21	2.180	1.06444	2.048
21 1/4			2.034
21 1/2			2.020
21 3/4			2.005
22	2.119	1.06410	1.991
22 1/4			1.977
22 1/2			1.963
22 3/4			1.948
23	2.057	1.06377	1.934
24	1.996	1.06343	
25	1.934	1.06310	

The second liquid investigated was a 27 percent by weight solution of glycerine in water. Two glycerine series were used to determine the effect of viscosity on the film thickness of liquids with the density, surface tension and Reynolds number held constant. Table 6 gives the physical property data for the glycerine solution.[32] It is to be noted in comparing Tables 5 and 6 that the density of water and of the glycerine solution are not significantly different. The effect of glycerine on the surface tension of water is negligible.[32] The essential difference is that the glycerine solution has a viscosity twice that of water. The two flow rates for the glycerine solution were:

1. Low-Middle Flow Rate Nominal Reynolds Number of 195
(This glycerine solution flow rate is comparable to the low-middle flow rate for water as the Reynolds numbers are essentially the same in both cases.)

2. High Flow Rate Nominal Reynolds Number of 640
(This glycerine solution flow rate is comparable to the high flow rate for water.)

The third liquid investigated, in two series, was a 0.1 percent by weight Aerosol-OT and water solution. This solution has essentially the same density and viscosity as water so its physical properties are summarized in Table 5 also. The surface tension of the Aerosol-OT solution was about 50 percent of the value for water. The two flow rates for the Aerosol-OT solution were:

1. Low Flow Rate Nominal Reynolds Number of 100
(This Aerosol solution flow rate is comparable to the low flow rate for water.)

2. High-Middle Flow Rate Nominal Reynolds Number of 455
(This Aerosol solution flow rate is comparable to the high-middle flow rate for water.)

Normalization of Experimental Data on Liquid Film Thickness

The theoretical Nusselt film thickness is dependant on the flow rate of the liquid, the width over which this flow takes place, and the temperature of the liquid film. The Reynolds number for a given flow, given by Equation 56, relates these three variables. In this equation Q is the peripheral flow rate: that is, the volumetric flow rate divided by the width of the liquid film, in this case the width being about 12 inches. Equation 57 in Appendix II gives the film thickness as a function of the Reynolds number and the kinematic viscosity.

The peripheral flow rate and the temperature of the run were not held constant throughout a given series but varied slightly from run to run. The theoretical Nusselt thickness was used as a normalizing parameter in order to compare the thickness in each test in a given series of runs with the corresponding thicknesses in the other runs of the series. This allowed the minor variations in the temperature and the flow rate to be corrected for. The effect of these minor variations on the Reynolds number and the theoretical Nusselt thickness are discussed in the next section.

The assumptions involved in the calculation of the theoretical Nusselt thickness and an outline of its development are presented in Appendix II. Thus, each experimental film thickness was divided by the Nusselt thickness for the particular flow rate and temperature at which the run was conducted. Hence, a film thickness of 1.00 is the same thickness as predicted by Nusselt; a thickness less than 1.00 is thinner than the theoretical thickness; and a thickness greater than 1.00 is thicker than the theoretical Nusselt thickness for a given flow rate and

temperature. The theoretical Nusselt thickness must be calculated for the particular flow configuration to determine the exact film thickness from the reported normalized thickness. This calculation is outlined in Appendix II.

Assuming that the actual film thickness reacts to changes in flow rate and temperature in the same manner that the theoretical thickness does, the variations in flow rate and temperature should not be reflected in the reported reduced actual thicknesses.

Method of Data Processing

For each of the experimental runs, the theoretical Nusselt film thickness and the Reynolds number were calculated using the basic data taken in the course of a run. The Reynolds numbers varied as much as 10 percent for the different experimental runs in a given series. In discussing a particular series, however, an average Reynolds number was used to simplify the discussion, this average is referred to as the nominal Reynolds number.

The time required to complete the experimental runs in a series ranged from several days to more than a week. The room temperature was not constant and varied from day to day as well as during any given day. Since small temperature changes were not considered to be important, the equipment was not designed for isothermal operation.

The Reynolds number is a function of temperature as well as velocity, and, in as much as the temperature of the liquid varied a little from day to day, the Reynolds number at constant liquid velocity was not constant over the time required to complete a series. No

attempt was made to adjust the flow rate so as to compensate for temperature changes, and the runs in a given series were made at approximately the same flow rate, or rather rotameter reading, instead of constant Reynolds number. The theoretical thickness, or Nusselt film thickness, should change in the same manner with temperature and flow rate as did the actual thickness, and so the ratio of the actual thickness to the Nusselt thickness is relatively insensitive to small temperature changes. This was the reason that the Nusselt thickness was used as a normalizing parameter.

As an example, take the low flow rate series with water. This series, with a nominal Reynolds number of 75, had actual Reynolds numbers as low as 66.[illegible] and as high as 79.3 with a numerical average value of 73.5. Hence, the Reynolds number for the low flow rate with water was 73.5 $\pm$ 9.2 percent. At this flow rate, the theoretical Nusselt thickness varied from a low of 1.88×10^{-2} cm. to a maximum of 2.04×10^{-2} cm. The average thickness was 1.97×10^{-2} cm. $\pm$ 4.6 percent. This case is most critical because at higher flow rates the percentage variation is smaller although the absolute values of the variations are larger. However, even with the ten percent change in Reynolds number the relative thickness should vary almost none at all from run to run. Looking at similar figures for the high flow rate with water, the Reynolds number varies from 653 to 731 with an average value of 680. Thus, the average Reynolds number if 680 $\pm$ 7.5 percent. However, looking at the film thicknesses, which is the most important consideration, they vary from 4.02×10^{-2} cm. to 4.20×10^{-2} cm. The average is 4.13×10^{-2} cm. $\pm$ 2.7 percent.

The maximum, minimum and average values of the Reynolds numbers and theoretical Nusselt thicknesses are given in Table 7 for each of the eight series. Also included in Table 7 is the percent deviation from the average of the maximum and the minimum values for the Reynolds numbers and the theoretical Nusselt thicknesses. With the exception of the low flow rates, the theoretical film thicknesses varied from one to three percent over the course of the ten or more runs which constituted a series. Thus, even if the variation in actual thickness with temperature and flow rate is not identical to the variation in theoretical thickness, the variation in reduced actual thickness from run to run should not be significant.

Method of Data Presentation

The film thickness data for each series of runs are presented in the next chapter graphically in figures similar to Figure 8. In each of these graphs, the film thickness at each position on the plate is represented by a horizontal line. The left end of the line segment represents the thinnest film detected in the run while the right end of the line segment represents the thickest film encountered. The short vertical tick in the center of the line segment is the numerical average of the 20 film thickness measurements made during each run at each position on the plate.

For some of the positions on the plate, there are two or more horizontal lines representing measured film thicknesses. In these cases, duplicate runs were made for comparison purposes. For convenience and ease of interpretation these duplicate runs are presented one

Table 7. Comparison of Reynolds Number and Theoretical Nusselt Thickness Variations Over an Experimental Series

	Reynolds Number				Nusselt Thickness cm. x 100			
Liquid	Max.	Min	Avg.	± %	Max.	Min.	Avg.	± %
Water	731	653	680	7.5	4.20	4.02	4.13	2.7
Water	491	443	466	5.4	3.70	3.58	3.64	1.7
Water	230	208	217	6.0	2.86	2.79	2.82	1.5
Water	79	67	74	9.2	2.04	1.88	1.97	4.6
Aerosol	476	439	454	4.9	3.36	3.23	3.26	3.1
Aerosol	105	92	99	6.7	2.06	1.89	1.96	5.1
Glycerine	652	623	639	2.5	5.88	5.77	5.82	1.1
Glycerine	199	187	193	3.1	3.97	3.87	3.92	1.3

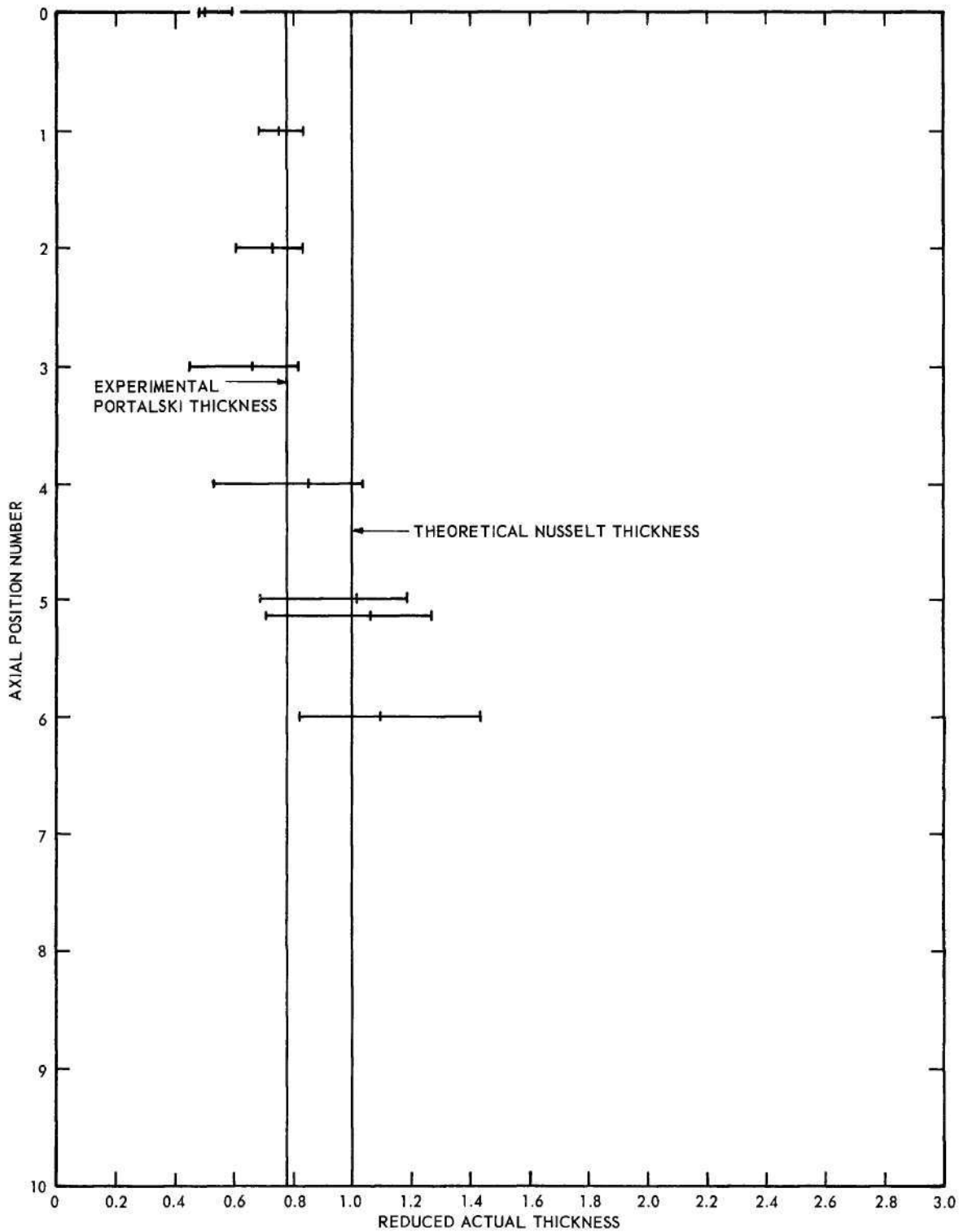

Figure 8. Mean Film Thickness and Wave Amplitude for Water at the Low Flow Rate.

slightly below the other. This is not to imply that the distance down the plate was different between these runs. This form of presentation is employed only for clarity and ease of comparison.

The abscissa of each graph represents the reduced actual thickness previously defined as the ratio of the actual thickness to the theoretically calculated Nusselt thickness at the given flow rate and temperature. These values vary from about 0.4 to 2.[illegible], or the actual thicknesses vary from about one-half the theoretical value to about two and one-half times the predicted value.

The numerical values from which the figures in the next chapter were drawn are presented in tabular form in Appendix III

Two additional vertical lines appear on the graphs. The first is merely a reference line at reduced actual thickness equal to one. This is, of course, an actual film thickness equal to the theoretically calculated thickness and is constant for all positions and all graphs or flow rates. The second line, the one to the left of the theoretical line, is the experimentally determined mean film thickness as reported by Portalski.[4] Portalski determined the mean film thickness on a flat plate 21 inches wide and seven feet long by means of a refined hold-up method for an extensive number of flow rates; these results agree with many of the later experiments which report mean thicknesses smaller than the theoretical prediction of Nusselt.

Finally, in the figures presenting the data for the glycerine and the Aerosol-OT solutions, there is an open-ended line below the closed line which corresponds to the data being presented in the figure The open-ended line is the film thickness spread recorded in the water runs

at the same nominal Reynolds number. This information is repeated from one of the previous figures showing water data to facilitate the comparison of the effect of viscosity and surface tension on film thickness. Where more than one water run was made, an averaged water line was drawn to reduce the possibly confusing effects of a multitude of data lines.

A total of 76 runs was made to determine the film thickness of vertically falling liquid films. Each run consisted of 20 measurements of film thickness for a total of over 1500 separate thickness determinations. The results are presented in three main categories: water, glycerine, and Aerosol-OT. The water data are presented in four groups, i.e., the four series which were made at different flow rates. The glycerine and Aerosol-OT solution data each are subdivided into two groups since two series at different flow rates were made for each of these liquids.

CHAPTER III

DISCUSSION OF EXPERIMENTAL RESULTS
PART I: FILM THICKNESS MEASUREMENTS

The film thickness of three different liquids flowing down a flat vertical plate was measured at various flow rates. The liquids were water, a glycerine-in-water solution and an Aerosol-OT-in-water solution. Each liquid will first be considered separately before comparisons between the different liquids are made.

Film Thickness of Water on a Vertical Flat Plate

The four separate series, or flow rates, at which water film thicknesses were determined are referred to as the low, the low-middle, the high-middle and the high flow rates.

Low Flow Rate

The series at the low flow rate with water--nominal Reynolds number of 75--consisted of eight runs. The runs were conducted at distances down the plate from 0 inch designated Position 0, i.e., the point of liquid introduction at the top edge of the plate, to Position 6 which was 5-3/4 inches down from the top edge. In addition, a duplicate run was made at Position 5 to check the reproducibility of the method. The data for this series are presented in Figure 8. At Position 0, the point of liquid introduction, the liquid film was considerably thinner than the Nusselt prediction. Moving 5/8 inch down the plate to the next test position, Position 1, the thickness increased

considerably but still was well below the Nusselt value. At this point, the thickness was close to, but less than, the mean value measured by Portalski. At the next position down the plate the film was slightly thinner; further down the plate, at Position 3, the film assumed its minimum average value. At this point the surface was well into the rippled region as evidenced by the greater spread of measured film thicknesses. Up to this position all of the average film thickness measurements were below both the predicted values of Nusselt and the measured values of Portalski. All of the measured points were below both of these values except seven of the maximum values which exceeded Portalski's mean measured thickness.

From this point on down the plate the trend in the measured thicknesses was an increase in the maximum, minimum and mean thickness in the direction of flow. The spread of thickness, i.e., the maximum measured thickness minus the minimum measured thickness, also increased. At Position 4 the mean measured thickness fell between Nusselt's prediction and Portalski's measured value while for the last two positions the mean value was greater than that given by either.

The two runs at Position 5 both gave essentially the same value for mean and minimum thickness while the maximum thickness differed slightly. The difference between the minimum values and the maximum values of thickness measured for these two runs fell within the limits of precision of the equipment. The reproducibility at this flow rate is within about four percent. The four percent error, due to the thinness of the 0.01 cm. film is the maximum encountered in this work.

Low-Middle Flow Rate

The Reynolds number for the low-middle flow rate was 215. Eight runs were made in this series in the seven positions from Position 0 to Position 6 with a duplicate run at Position 3. These were the same positions as those used in the low-flow-rate tests except that the duplicate run was made at Position 3 rather than at Position 5. The data are presented in Figure 9. At Position 0, or the top edge of the plate, the film thickness was slightly larger than the Nusselt theoretical thickness and considerably above the mean measured thickness reported by Portalski. This is a marked change from the condition at the entrance in the low flow rate where the thickness was considerably thinner than the mean value of Portalski at the entrance but rapidly thickened to this mean value a short distance down the plate. For the low-middle flow rate, the initially thicker film immediately necked down to Portalski's measured mean at Position 1 and held essentially the same thickness down to Position 2. While this thickness was the same as the mean thickness measured by Portalski it was 12 percent less than the Nusselt thickness for this flow condition.

Between Position 2 and Position 3 a marked transition appeared. The mean thickness increased from 90 percent to 120 percent of theoretical and the spread of thickness, or wave amplitude, was about four times as large as in the previous position. From here on down the plate the trend was constant. The mean thickness was well above both previously measured mean values and above the Nusselt thickness. While the mean value slightly increased farther down the plate, there was no great change from one position to the next. In each position, the minimum

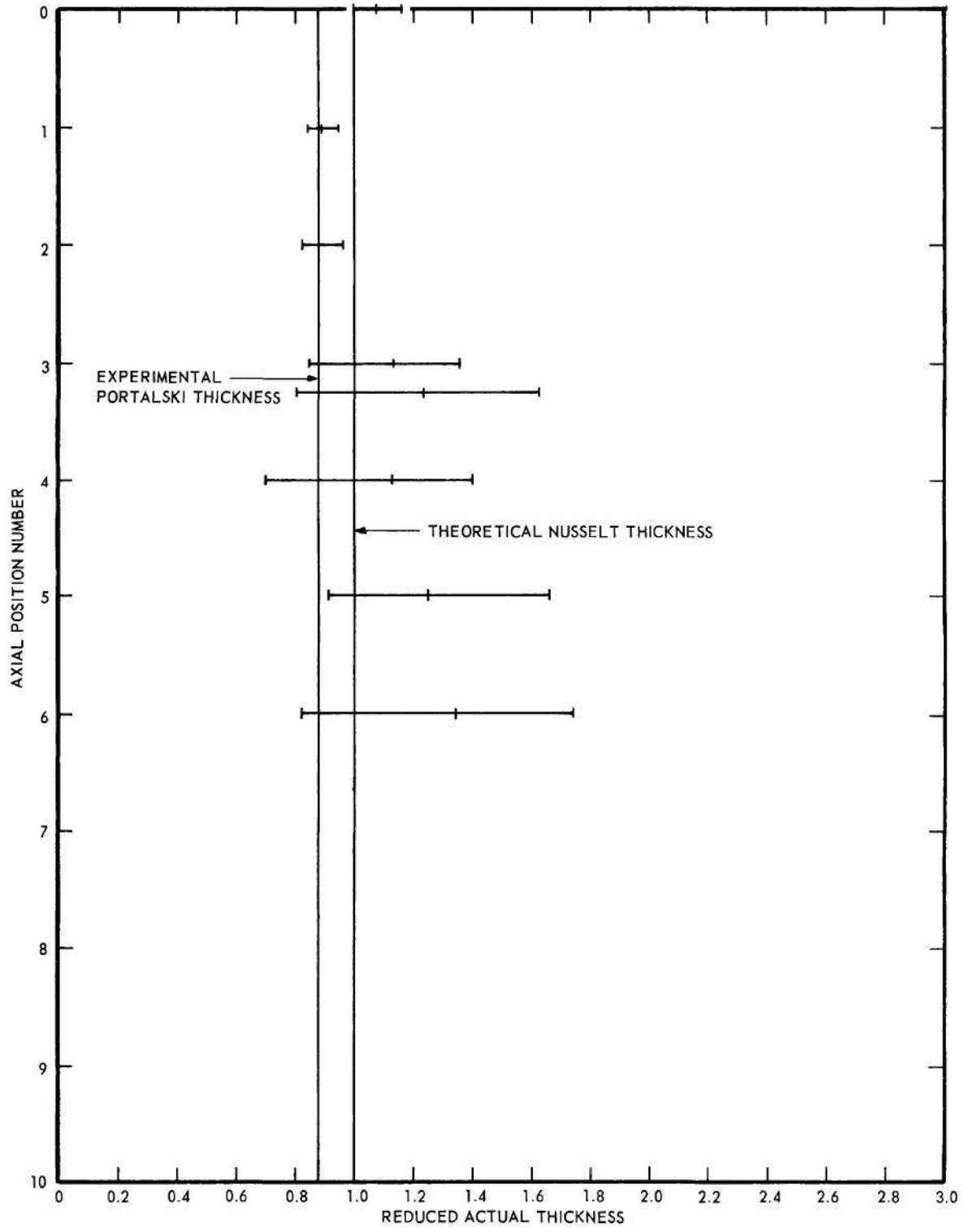

Figure 9. Mean Film Thickness and Wave Amplitude for Water at the Low-Middle Flow Rate.

thickness was about 80 percent of theoretical while the maximum thickness went up to 160 percent of theoretical. In all of these positions (Positions 3 to 6) the mean thickness was about 20 percent greater than the Nusselt thickness and about 30 percent greater than Portalski's experimental mean thickness.

The larger initial thickness in the low-middle flow rate rapidly decreased to the same mean film thickness as reported previously by Portalski. At this point the spread of thickness was relatively small; however, after passing this point the spread of data rapidly increased almost as a step function and at the same time the mean film thickness similarly increased. From this point on down the plate the mean film thickness gradually increased while the minimum and the maximum thickness changed very little.

The significance of the increase in the spread of the data will be discussed in more detail later. It may be noted, however, that this definite change in thickness variation is related to the point at which ripples or waves appear on the liquid surface.

High-Middle Flow Rate

The Reynolds number for the high-middle flow rate was 460. This series consisted of eleven runs which were made starting at Position 0 and continuing 7-3/4 inches down the plate to Position 8. This was two positions farther down the plate than were used with the low and the low-middle flow-rate series. The reason for going farther down the plate was that the smooth flow region covered a larger portion of the flow plate and the area of interest, the region of initial rippled flow, occurred farther down the plate. The results of the measurements for

this flow rate are shown in Figure 10.

At Position 0, for the high-middle flow rate, the initial thickness of the film was 70 percent greater than the Nusselt thickness. This compared with a thickness ten percent greater than the theoretical for the low-middle flow rate and 50 percent thinner than the theoretical for the low flow rate. As may be noted from the next section, this trend of increasing initial thickness with increasing flow rate was continued in the high flow rate which had an initial thickness 120 percent greater than the theoretical thickness. The initial thickness at the high-middle flow rate dropped to the Nusselt thickness at Position 1. The thinning continued to Position 3 where the relative thickness was 0.92, i.e., a film thickness eight percent less than the Nusselt prediction.

At Position 3 the spread of data began to increase. Position 4 had a definitely larger wave amplitude in comparison to Position 3 than did Position 3 in comparison to Position 2. However, the mean film thickness was still decreasing and reached its minimum of 0.88 at Position 4. This behavior was the same as demonstrated in the low-middle flow rate; however, it occurred one position farther down the plate in the high-middle flow rate. From Position 4 on down the plate the minimum measured film thickness held relatively constant while the mean and maximum measured film thicknesses slowly but steadily increased. Similar to the low-middle flow rate, the minimum film thickness was about 0.8 but the mean thickness was larger, 1.5 compared to 1.3. The maximum film thicknesses in the high-middle flow rate were considerably larger than for the low-middle flow rate, 2.0 compared to 1.6.

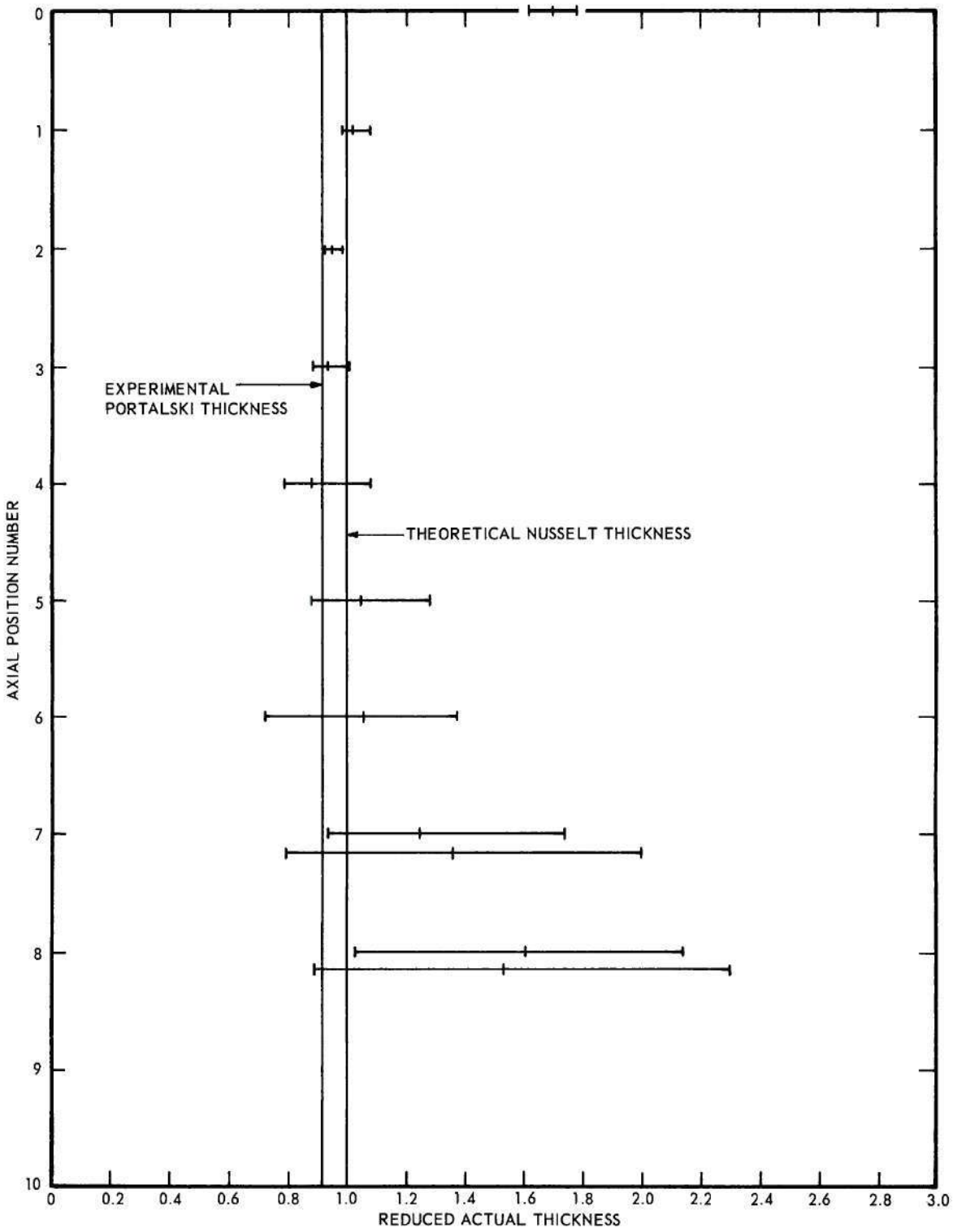

Figure 10. Mean Film Thickness and Wave Amplitude for Water at the High-Middle Flow Rate.

As in the low-middle flow rate, a large initial thickness decreased to a minimum thickness below the theoretical thickness. When the minimum mean thickness was reached, or perhaps one position before this, the spread of thicknesses increased. This indicated that the smooth surface of the liquid was undergoing a change in flow configuration. From this point down the plate, the thicknesses and the data spread increased.

High Flow Rate

The Reynolds number for the final water series, the highest flow rate investigated, was 680. This series consisted of 14 runs at the same positions covered in the high-middle flow rate series with three runs being made at each of the last two positions, Positions 7 and 8, and two runs being made at Position 3. The data are shown in Figure 11.

The pattern was the same as in the two previous series. The entrance thickness was considerably greater than the theoretical thickness but it decreased as it traveled down the plate reaching a minimum reduced thickness of 0.90 at Position 4. At this point a slight increase in mean film thickness and data spread began but there was no large change in either quantity until Position 7 was reached. At Position 7 there was a marked increase in both quantities.

There was a significant difference in the three runs made at Position 7. In the first of the three the mean film thickness and the data spread are both considerably less than in the following two. All three runs were made at the same position but the slight changes in the flow conditions were such that there was considerably less surface fluctuation in the first run than in the other two runs. This indicates that the amount of surface variation, or rippling, affects the

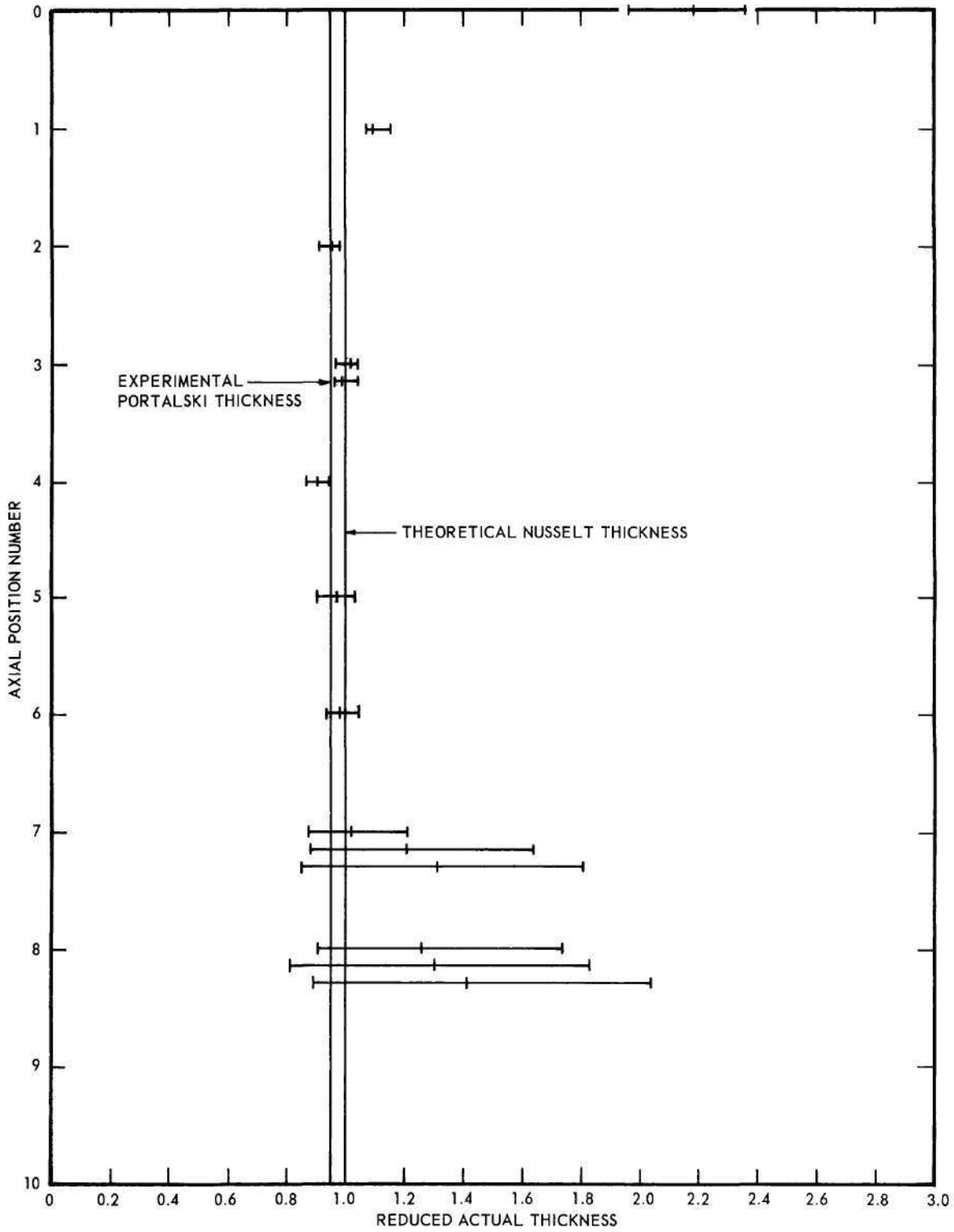

Figure 11. Mean Film Thickness and Wave Amplitude for Water at the High Flow Rate.

apparent film thickness. The remaining runs in this series show the type of variation that existed when the rippled surface was measured.

General Comments on the Four Water Series

At all but the low flow rate the initial film thickness was greater than the theoretical Nusselt film thickness. It was also greater than Portalski's experimental mean film thickness and greater than the minimum mean film thickness as found in this work. This could be caused by the method of introduction of the water onto the flat plate which introduced a velocity component normal to the plate. This would tend to increase the film thickness, particularly at the higher flow rates. At the low flow rate, this effect was negligible. While the initial thickness was relatively large, the film thickness rapidly decreased to a much thinner film after traveling a short distance down the plate. The minimum film thickness found for each series was about 90 percent of the theoretical thickness except in the low flow rate series. After the minimum film thickness had been reached the film thickness started to increase. The increase was gradual over a short distance and then a quite sudden increase was observed. This sudden increase in film thickness was related to a similar sudden increase in data spread or wave amplitude. The increase in surface fluctuations or data spread occurred at the point of minimum thickness or just below this point.

Wave formation occurred with, or immediately after, the minimum film thickness had been attained in all series if a short entrance length is discounted in the low flow rate series. Also it may be noted that the minimum thickness attained in the three higher flow rates was

between 0.88 and 0.90. In all cases a mean film thickness thinner than that predicted by Nusselt was found. Also, this minimum mean film thickness was thinner than the mean film thickness measured by Portalski; although for one series, the low-middle flow rate, it was the same.

Film Thickness of a Glycerine Solution on a Vertical Flat Plate

Measurements were made at two flow rates (two series) with a 27 percent by weight glycerine-in-water solution. The flow rates were at about the same Reynolds numbers as the low-middle and the high flow rates of the water series. Comparison of the Reynolds numbers at which the two glycerine series were run with the corresponding water series is shown in Table 6. The lower flow rate--a Reynolds number of 193--used with glycerine, will be compared to the low-middle flow rate with water, nominal Reynolds number of 217. This flow rate will be referred to as the low-middle glycerine flow rate. The higher velocity test of the two glycerine solution series corresponded to an average Reynolds number of 639. This flow rate series, which will be called the high glycerine flow rate, will be compared to the high flow rate of water having a Reynolds number of 680.

The glycerine solution had approximately twice the viscosity of pure water. There was essentially no difference in either the density or the surface tension between the glycerine solution and water. The 100 percent increase in viscosity of this solution over water resulted in a corresponding 50 percent increase in theoretical film thickness.

The nomenclature on the figures which present the glycerine film thickness data is the same as that used in the previous water series

with one addition. Under each closed-end horizontal line, which represents the minimum and the maximum film thicknesses for the glycerine films, is an open-end line. This open-end line represents the measurements for the water at about the same Reynolds number. This allows comparison between the films of the glycerine solutions and the water films.

Low-Middle Flow Rate

The Reynolds number for the low-middle glycerine flow rate was 193. This series consisted of six runs. The runs were made from Position 0 through Position 5. The data are summarized in Figure 12.

At Position 0 the initial film thickness was about 20 percent greater than the Nusselt thickness. The film rapidly thinned and at Position 1 reached its minimum thickness, about 95 percent of theoretical. At Position 2 the film thickness slightly increased to about the theoretical thickness, and at this position the wave amplitude was noticeably larger in comparison to the amplitude at the previous position. The final three positions, Position 3 to Position 5, showed a continuing increase in data spread, or amplitude, and film thickness.

In both the water series and the glycerine series the theoretical Nusselt thickness was used as a normalizing parameter. This means that all of the thicknesses are relative to a thickness of 1.0 but this relative thickness of 1.0 corresponds to a film larger in magnitude for the glycerine solutions than for water because, with the increased viscosity, there is an increased theoretical thickness.

Comparison of the glycerine and the water series indicated that at the entrance and first few positions the glycerine film is relatively

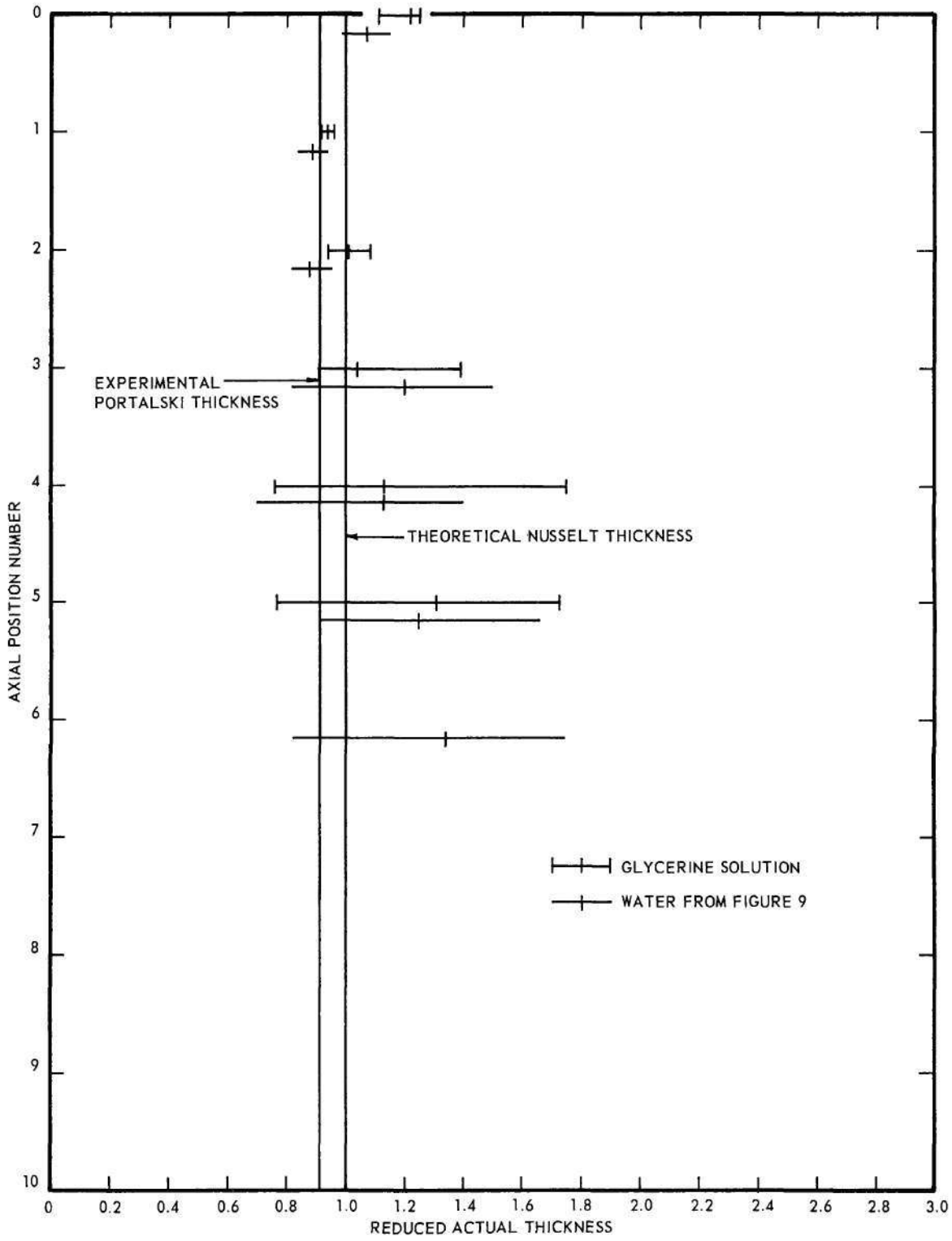

Figure 12. Mean Film Thickness and Wave Amplitude for the Glycerine Solution at the Low-Middle Flow Rate.

thicker than the water film. The reduced thickness for glycerine remained larger than that for water through Position 2 which was the minimum mean thickness for both liquids. At Position 3 the reduced thickness of the water film was larger than the reduced thickness of the glycerine film, but from that point on down the plate the glycerine maintained a thicker film than the water. Both the glycerine and water series at the low-middle flow rate reached their minimum thickness at the same position. Both also showed a marked increase in the spread of the data at the same position and to about the same extent.

High Flow Rate

The Reynolds number for the high glycerine flow rate was 640. This series consisted of nine runs made from Position 0 to Position 8. This series, which corresponded to the high flow rate with water, is summarized in Figure 13. The water data, represented by the open-end lines, were taken from Figure 11.

At Position 0 the film thickness was almost three times the Nusselt thickness. This was the thickest film encountered in the experimental investigation. At Position 1, only 5/8 inch down the plate, the thickness fell to 1.4 in a fashion typical of the previous series. The film thickness decreased on down the plate until at Position 5 it reached its minimum thickness. This thickness was slightly higher than the predicted theoretical thickness and was the only instance in which the minimum thickness exceeded the theoretical thickness. At Position 6 a slight increase occurred in data spread and film thickness, and at Position 7 a drastic increase in both quantities was found. At the final position, Position 8, the mean film thickness

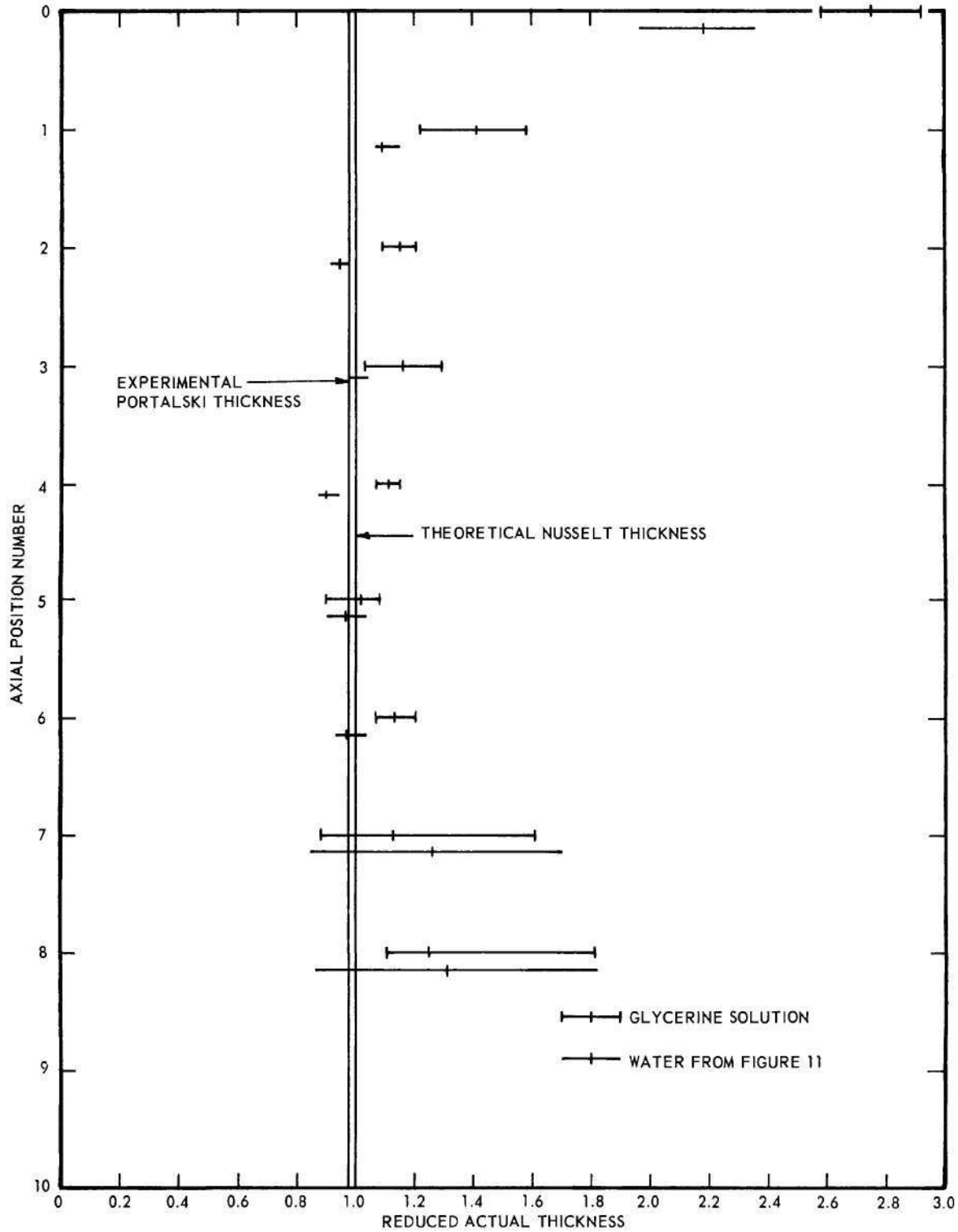

Figure 13. Mean Film Thickness and Wave Amplitude for the Glycerine Solution at the High Flow Rate.

increased slightly from the previous value at Position 7.

As in the low-middle glycerine flow rate, the glycerine films remained thicker than the comparable water films until the minimum film thickness was reached. This trend continued until the position was reached at which the first drastic increase in data spread and film thickness occurred. For this one position the water films were relatively thicker than the glycerine films, but the glycerine films then became relatively larger again as the measurements were made farther down the plate.

General Comments on the Two Glycerine Series

In general the results for the water and glycerine series were similar, although there was one significant difference. Both liquids entered as thick films and rapidly thinned to values which closely approximated the theoretical. From here on down a gradual decrease in thickness occurred until a minimum film thickness was reached. At this minimum or shortly below, the data spread began to increase which indicated surface fluctuations. Not only did this same pattern appear for both liquids but it appeared at the same position on the plate. Finally, after this minimum had been reached and wave motion had begun, the mean film thicknesses increased and the data spread became greater because the film surface was no longer smooth but was in rapid motion. The significant difference between the water series and the glycerine series was that the glycerine was relatively thicker than the water up to and through the film thickness minimum and the beginning of the pronounced increase in data spread. Then in both glycerine series the next position, the first position with a significantly higher data

spread, showed the water film thickness relatively larger than the glycerine film thickness. The remainder of the runs seem to indicate that this difference in relative film thickness is not as pronounced between water and glycerine in the region where large scale rippling occurs.

Film Thickness of an Aerosol-OT Solution on a Vertical Flat Plate

Two series of runs were made in which a surface-active agent, Aerosol-OT, was dissolved in water. The surface-active agent reduced the surface tension to about one-half of its previous value but had essentially no effect on the viscosity or the density of the test solution at the concentration used, 0.1 percent by weight. The flow rates at which the two Aerosol-OT series were conducted were comparable to the low and the high-middle flow rates with pure water.

In Table 6 the Reynolds numbers at which the two Aerosol-OT series were run are compared to the corresponding water series. The low Aerosol-OT flow rate at an average Reynolds number of 98.6 is compared with the low flow rate water series having an average Reynolds number of 73.5, which is somewhat lower than the Aerosol-OT series but close enough for comparison purposes. The high-middle flow rate with Aerosol-OT is comparable to the high-middle flow rate with water. The Aerosol-OT had an average Reynolds number of 454 while the average value for the water series was 462.

The nomenclature on the figures summarizing the film thickness data for the Aerosol-OT series is the same as that used in previous sections. The closed-end line represents the Aerosol-OT thickness

spread while the open-end line represents the thickness spread for the pure water series, at a comparable flow rate and at the same position.

Low Flow Rate

The series at the low Aerosol flow rate consisted of eight runs which extended through Position 9. This series is summarized in Figure 14. The water data on Figure 14, the open-end lines, were taken from Figure 8.

At Position 0 the film thickness was considerably less than the Nusselt thickness. At Position 1 the thickness had increased but was still less than the theoretical. At Position 2 the thickness had increased again slightly, to its maximum value, which was three percent greater than the Nusselt thickness. From this point on down the plate, the film thickness was essentially constant. The average film thickness was slightly less than the Nusselt value for the most part although in two of the six positions the films were slightly thicker. Two characteristics may be noted; first, there was relatively little spread of data when the Aerosol solution was used, and second, there was no increase in average film thickness down the plate. The first point concurs with visual observation which indicated that there was no rippling on the film surface and the second appears to be a result of the first.

High-Middle Flow Rate

The series at the high-middle Aerosol flow rate consisted of eight runs at the same positions as the low Aerosol flow rate. The series was run at a nominal Reynolds number of 454, in order to compare the results with the high middle flow rate for water with a Reynolds number of 462. The Aerosol series is summarized in Figure 15 where the

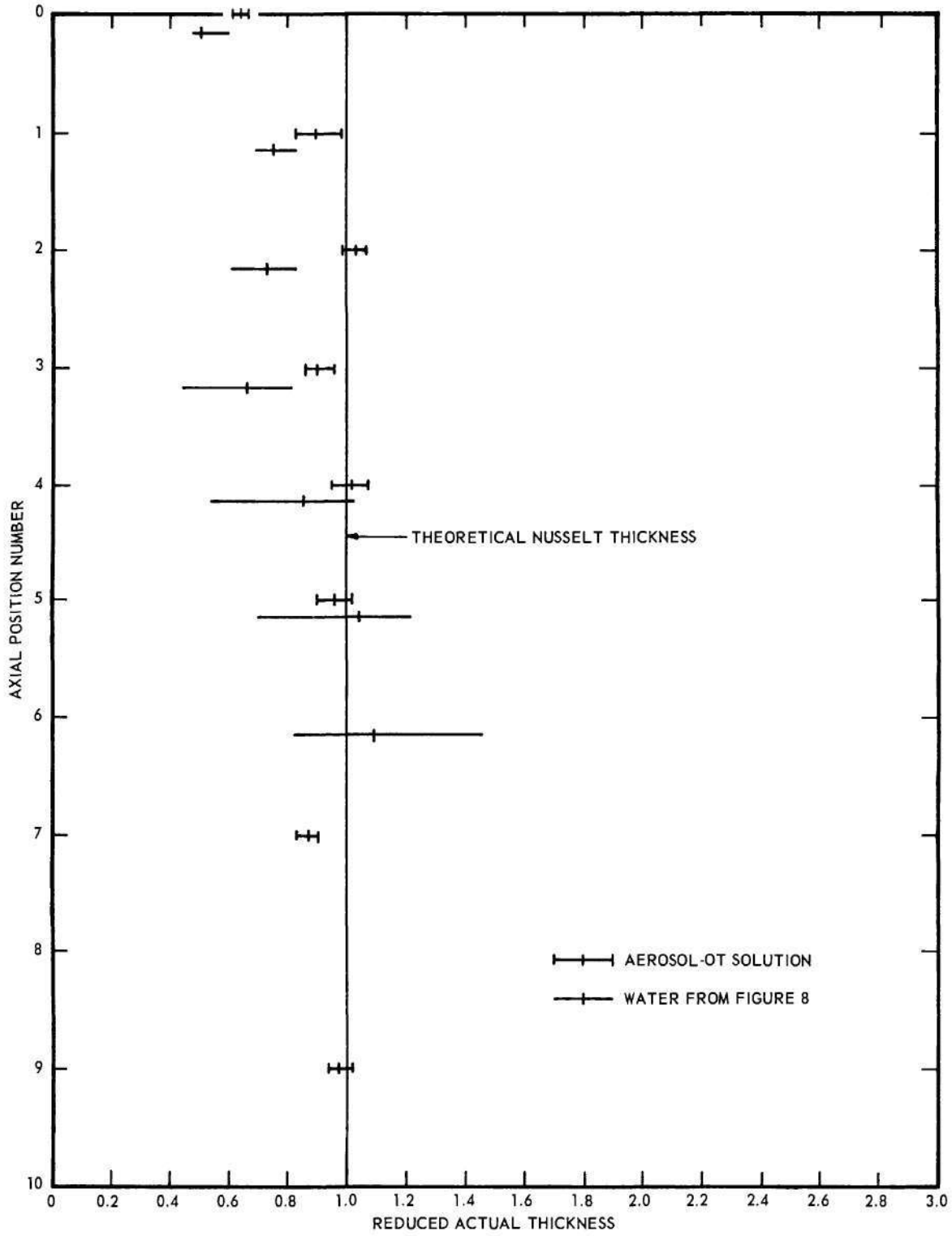

Figure 14. Mean Film Thickness and Wave Amplitude for the Aerosol-OT Solution at the Low Flow Rate.

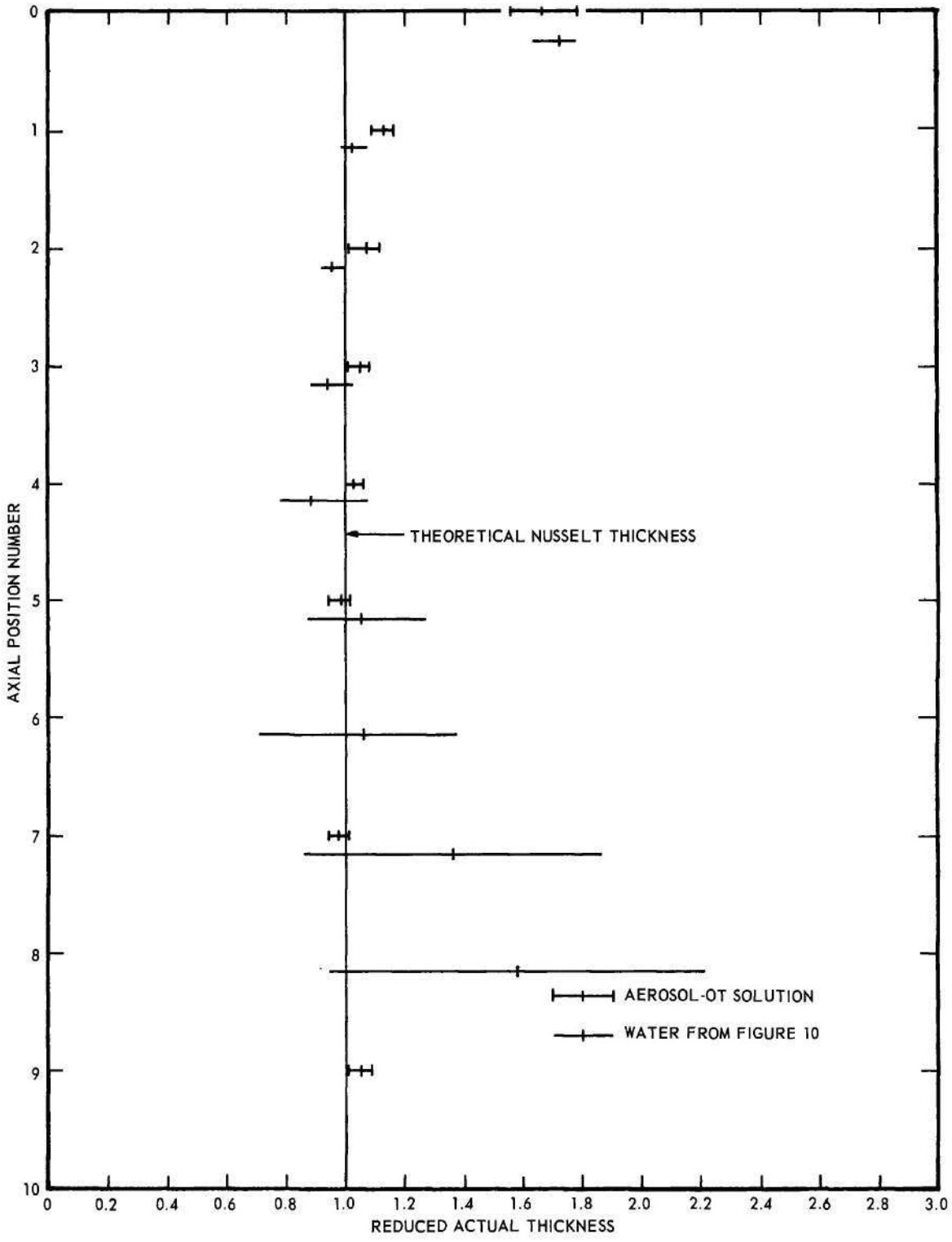

Figure 15. Mean Film Thickness and Wave Amplitude for the Aerosol-OT Solution at the High-Middle Flow Rate.

comparable water data from Figure 10 is also shown.

At Position 0 the Aerosol-OT solution entered at about the same thickness as for the comparable water series, approximately 1.7 times the predicted thickness. As with water, the thickness decreased rapidly at the next position and then continued to decrease slowly in thickness to Position 7 where it reached its minimum value which was only slightly below the theoretical Nusselt thickness. Only one measurement was taken below this point, at Position 9, and the thickness was slightly more than the minimum thickness. From this position on down the plate no rippling was visually observed.

General Comments on the Two Aerosol-OT Series

There are strong similarities as well as marked differences in the Aerosol-OT series compared to the water series run with approximately the same Reynolds numbers. The initial film thickness in the low flow rate water series was less than the theoretical thickness, whereas for all other series it was greater. This characteristic was duplicated, almost to the same reduced actual thicknesses, in the Aerosol-OT series. With water, the initial film thickness went through a minimum value as it traveled down the plate. The same behavior was noted in the Aerosol-OT series. With water, once the minimum thickness was attained, a marked increase in film thickness and data spread was detected as measurements were continued down the plate, whereas with Aerosol-OT no appreciable increase in the data spread was noticed, but there was a slight increase in film thickness after the minimum was reached. Essentially the same trends were noted with both liquids; especially if the discussion is restricted to the region of the plate above the position of rippling in

the water series. In this case the Aerosol-OT data, point for point, duplicate the water data except that at each point the Aerosol-OT films were thicker than the water films. At a certain point down the plate in the water series a minimum thickness was reached, while beyond this point the film thickness and the data spread both increased greatly. With the Aerosol-OT series, the minimum was also reached but somewhat further down the plate than with water.

The minimum was followed, however, by only a slight increase in film thickness and no increase in the spread of the data. This was because the Aerosol-OT suppressed the rippling. The thickness of the water film increased above that of the Aerosol-OT film once rippling occurred on the water surface.

PART II: DISTANCE BEFORE WAVE INCEPTION

The stability of the laminar flow of a liquid layer has been a topic of considerable interest in recent years. Kapitza[33] was one of the first to attack analytically this problem. The major result of this analysis was the prediction of a critical Reynolds number below which the flow of the liquid was stable and above which the flow was unstable or rippling. Later the analysis was extended by Portalski[34] with the same conclusion, i.e., there exists a critical Reynolds number below which the liquid flow is stable. The stability problem was more rigorously formulated by Yih[8] in 1955 and, by means of numerical computation, the equations were solved resulting in a critical Reynolds number of six below which stable flow existed. This, however, did not completely agree with some experimental observations which indicated surface disturbances at practically all Reynolds numbers and in particular at some Reynolds numbers less than six.

In 1957 Benjamin,[7] in an analysis based on a variation of Yih's formulation, performed a new calculation with the important difference that neutral stability curves were obtained analytically rather than numerically. These calculations established that free-surface flow down a vertical plane is unstable for all finite Reynolds numbers. In a further refinement of technique, Yih[35] dispelled the discrepancy between his and Benjamin's work and further showed that, for all Reynolds numbers, the free-surface flow down a flat plate was unstable. Benjamin showed a relationship between the unstable flow and the phenomena of

rippling and explained, by means of a calculated amplification factor, that rippling was present at all flow rates but that at extremely low flow rates the waves were too small to be observed. This implies that ripples are always present on falling liquid films but that they are not always visible.

The Line of Wave Inception

From the literature and in the course of this investigation it was noted that for a short distance below the liquid distributor there exists a section of the film which appears to be wave free. At a given distance from the distributor, there is a sudden appearance of ripples. The position at which the rippled flow, or wave motion, appears is at a constant distance from the top of the plate across the entire flow width. The length of wave-free flow is dependent on the flow rate of the liquid. This wave-free distance of flow may be varied by changing the flow rate.

In this investigation the distance down the plate from the point of liquid entrance to the inception of waves on the surface was measured, and rippling was found to be present at all locations farther down the plate. This distance, or length down the plate, is called the distance before wave inception. This position at which rippling commences is not to be confused with the stability criteria which states that the vertical flow of a liquid is unstable at all flow rates, or at least at all flow rates above a certain critical Reynolds number. Even liquids at high laminar flow rates (above the critical Reynolds number) which all theories term unstable have a certain length below the liquid entrance which is relatively free of any surface disturbances before rippling begins.

Prior Experimental Reports

Tailby and Portalski[36] have measured the distance before wave inception by two methods. The first was visual observation of the point at which the ripples began; the second, photographing the flow and determining from the resultant photograph the point at which ripples began. Portalski found that increasing the flow rate increased the distance before wave inception. The Reynolds numbers investigated in that work varied from a low of about 70 to well within the turbulent region for water.

A second report of experimentally determined distance before wave inception is due to Batt,[10] who used a light-absorption technique with two light beams, one passing through the upper portion of the flow plane where the flow was reasonably smooth and a second passing through a lower portion of the flow plane. Apparently, the first beam was used to nullify the effect of the average amount of light which passed through the water and to magnify the effect of the changing liquid depth in the rippled flow region. Rippling was determined by this method when the net output of the light collection system began to fluctuate.

The data reported by Batt parallels that of Portalski but indicates a slightly shorter distance before wave inception in the flow rates investigated by both. However, for flow rates lower than a Reynolds number of 70, Batt reports an increasing distance before wave inception as the flow rate is decreased.

Present Measurement Technique

The distance before wave inception was determined in this inves-

tigation by analyzing the instantaneous film thickness data. For the upper positions on the plate where there is no wavy flow, the film thickness should not vary at a given position on the plate. This would be the case if the film surface were absolutely smooth and the measurements were all exact. However, there is a certain limit to the precision of the equipment. This limit introduces a small spread in the data. Also, if the film flow is unstable at all Reynolds numbers, some fluctuations in film surface would be expected. Nevertheless, it was noted that in the upper sections of the flow plate, the spread of data, i.e., the maximum measured film thickness minus the minimum measured film thickness at a given point, was fairly small. This difference represents the wave amplitude at the given position on the plate. This wave amplitude, while quite small at the top of the plate where no ripples could be visually detected, was much greater at the bottom of the plate where it was obvious that the flow was irregular and uneven.

It is proposed that the distance before wave inception be defined as the distance to that section of the flow plate in which there occurs a relatively sudden increase in wave amplitude over a relatively short distance of travel down the plate.

This criteria was applied to the data of this investigation and the results then compared with the data of Portalski and Batt. In Table 8 the wave amplitudes for eight series of runs are shown. The wave amplitudes reported are reduced amplitudes and are dimensionless, that is, they are defined by the relation,

$$\text{Reduced Amplitude} = \frac{\text{Maximum Thickness - Minimum Thickness}}{\text{Theoretical Nusselt Thickness}}$$

Table 8. Wave Amplitudes as a Function of Flow Rate, Position, and Type of Liquid

Position Number	Wave Amplitudes (Reduced Actual Thicknesses)							
	Water				Glycerine Solution		Aerosol-OT Solution	
	Low	Low-Middle	High-Middle	High	Low-Middle	High	Low	High-Middle
0	0.11	0.15	0.15	0.39	0.09	0.33	0.04	0.22
1	0.14	0.10	0.08	0.08	0.05	0.36	0.15	0.07
2	0.22	0.13	0.06	0.07	0.15	0.11	0.07	0.09
3	0.36	0.52 0.82	0.12	0.07 0.08	0.48	0.26	0.10	0.07
4	0.50	0.70	0.28	0.07	0.99	0.07	0.11	0.05
5	0.49 0.56	0.75	0.40	0.13	0.96	0.18	0.11	0.07
6	0.62	0.92	0.66	0.10	----	0.12	----	----
7	----	----	0.80 1.21	0.34 0.76 0.96	----	0.71	0.06	0.06
8	----	----	1.11 1.41	0.83 1.02 1.05	----	0.70	----	----
9	----	----	----	----	----	----	0.07	0.08

The behavior of the wave amplitudes shown in Table 8 was similar for the four water series and the two glycerine series. There was a decrease in wave amplitude until a minimum amplitude was reached at a given position on the plate. After this position, which was a function of the flow rate and of the liquid flowing, was reached, further travel down the plate resulted in an increase in the wave amplitude.

The length assigned as the distance before wave inception depended on a subjective choice and also was dependant on the spacing of the positions at which the film thickness measurements were made. In this work the distance before wave inception was used which would give the maximum value to this length. The method employed was to pick the position number which beyond any question or doubt was as far, or farther, down the plate than the position at which the wave amplitude had drastically increased. This position and the position up the plate then were stated as the range which bracketed the length at which rippling had definitely occurred. The distance before wave inception was then the maximum length through which a liquid could flow with a relatively smooth surface.

Wave amplitudes and film thicknesses both show a similar trend as a function of position number. Both have a minimum and then a period of slow increase followed by a rapid increase. The wave amplitudes in Table 8, in most cases, decrease to a minimum amplitude of between 0.05 and 0.10 after which an increase in amplitude occurs. This minimum amplitude was much more constant for the given liquids over the flow rates studied than the minimum film thicknesses.

The wave amplitudes for the Aerosol-OT solution, for which it

has been often reported that rippling does not occur, are essentially constant on down the plate. The inability to assign a distance before wave inception verified previous reports that a surface-active agent will damp out surface fluctuations.

The wave amplitudes for the glycerine solution series show the same behavior as the wave amplitudes for the water series which were run at comparable flow rates. For both glycerine solution series the distance before wave inception was the same as for the similar flow-rate water series.

Comparison of Results

A comparison of this method for determining the distance before wave inception to that of Portalski and Batt is shown in Table 9. It may be seen that all distances before wave inception obtained by this method are less than the distances reported by Portalski for the same flow rates, which indicates that this method is more sensitive than either the visual or the photographic method. The data of Batt fall very near the value given by this method. This is not unexpected because he used the same phenomena (the surface fluctuation of the film) to determine the distance before wave inception. Batt measured it in a different manner, however.

Recently Bruley[37] calculated the entrance region characteristics for liquid films flowing over a vertical, flat surface by numerically solving certain two-dimensional boundary layer and continuity equations. The inlet length was defined as the distance from the leading edge at which the surface velocity reached 99.99 percent of its asymptotic

Table 9. Comparison of the Different Values of the Distance Before Wave Inception

Experimentor Method	Distance Before Wave Inception (cm.)			
	Water			
	Low	Low-Middle	High-Middle	High
Portalski Visual	4.8	9.5	14	17
Portalski Photographic	---	7.7	13	16
Batt Light Absorption	4.2	6.0	---	---
Present Work Thickness Measurement	1.6-4.4	4.4-6.7	6.7-9.5	14.6-16.9
Bruley Theoretical	----	0.7	2.3	4.1
	Glycerine			
		Low	High	
Portalski		8.5	16.5	
Present Work		4.4-6.7	14.6-16.9	

terminal flow velocity. A main assumption was that the entrance velocity profile was flat. Bruley's formulation made no provision for instability, so in the mathematical model rippled flow does not exist. The entrance lengths calculated by Bruley are given in Table 9, and in all cases the lengths fell well below the experimentally observed distance before wave inception. This may be an unfair comparison in that the calculation does not attempt to predict instability but only predict the length it takes for the entering liquid to assume a substantially steady-state profile. According to the film thickness data of this investigation the liquid film continued to decrease in thickness well after the entrance length predicted by Bruley. This may be an indication of the complexity of the overall vertical flow problem.

PART III: REGARDING THE POSSIBILITY OF SLIP ON A VERTICAL WALL BY A NEWTONIAN LIQUID

Throughout the experimental program the film thickness, particularly at the lower flow rates, was significantly different from the calculated Nusselt thickness. The films were all thinner than theoretical above the position on the plate where pronounced rippling began. Within the rippled region it might be argued that, since the surface is so irregular and the flow patterns so disturbed, no particular correlation with the Nusselt theory could be expected.

Among the factors acting on a thin liquid film which could cause it to be thinner than the Nusselt thickness might be the movement down the plate of the liquid surface which is in contact with the plate. In other words, if the liquid slips at the wall, a thinner film would be expected at a given mass flow rate. In order to evaluate the effect of slip at the solid-liquid interface, one boundary condition used in the development of the equations for Nusselt flow was modified to include a slip velocity at the wall.

The development accounting for the effect of slip at the wall is identical to that reported in the Appendix II through Equation (42). The nomenclature is shown in Figure 16, so that changes from the nomenclature used in Appendix II may be noted.

Theoretical Derivation

Equation (42) developed in Appendix II is

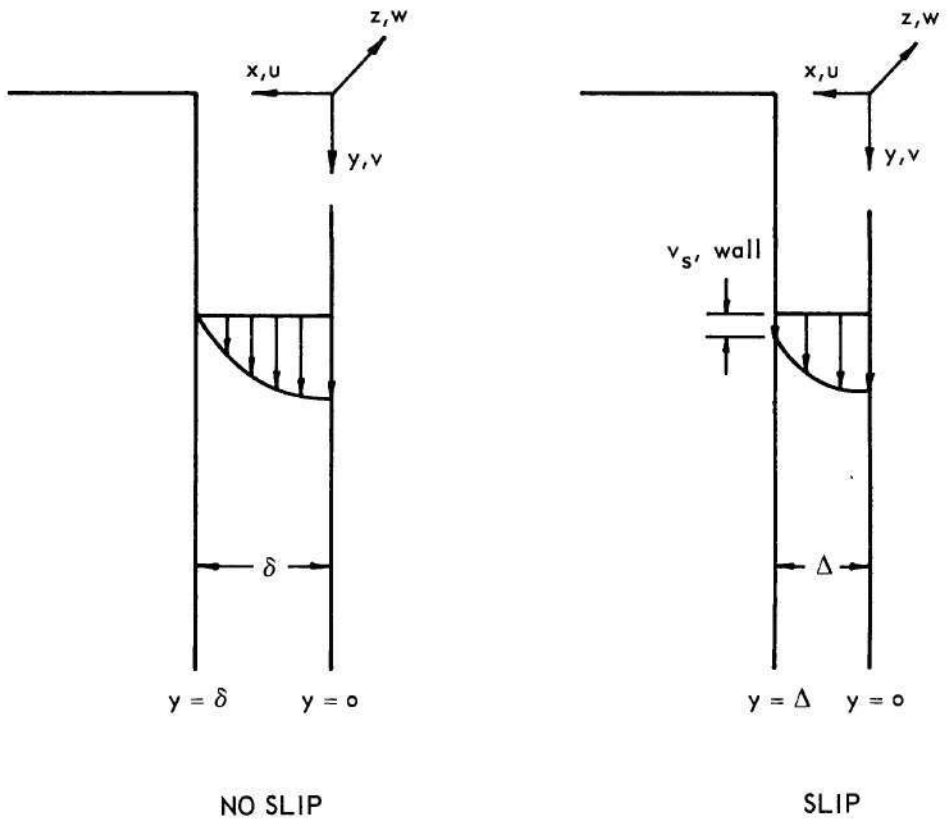

Figure 16. Comparison of the Coordinate Systems Used in the Slip and in the Nusselt Derivations.

$$v = \frac{-gy^2}{2\nu} + C_2 \tag{1}$$

The boundary condition at the wall will take the form

$$v_s = \frac{-\mu \frac{dv_s}{dy}}{\beta} \text{ at } y = \Delta \tag{2}$$

The subscript s used on the velocity term refers to the case where slip occurs at the wall. When slip is considered, the film thickness will be referred to as Δ rather than δ as in the Nusselt case. The coefficient of slip is a constant, β. Except for one boundary condition, that at the wall, the assumptions are identical to those made by Nusselt.

Equation (1) becomes

$$v_s = \frac{-gy^2}{2\nu} + C_2 \tag{3}$$

From Equation (2), the boundary condition is

$$\frac{dv_{s,wall}}{dy} = \frac{-\beta v_{s,wall}}{\mu} \text{ at } y = \Delta \tag{4}$$

Differentiating Equation (3) gives

$$\frac{dv_s}{dy} - \frac{-gy}{\nu} \tag{5}$$

At the wall

$$\frac{dv_{s,wall}}{dy} = \frac{-g\Delta}{\nu} \text{ at } y = \Delta \tag{6}$$

Solving Equations (4) and (6) for the wall velocity gives

$$v_{s,wall} = \frac{g\rho\Delta}{\beta} \tag{7}$$

This condition, the boundary condition, is now applied to Equation (3) to evaluate the integration constant C_2. The result is

$$C_2 = \frac{g\rho\Delta}{\beta} + \frac{g\Delta^2}{2\nu} \tag{8}$$

This value for C_2 is substituted into Equation (3) and gives, on rearrangement, the velocity profile for slip flow

$$v_s = \frac{g\Delta^2}{2\nu}\left[1 + \frac{2\mu}{\beta\Delta} - \left(\frac{y}{\Delta}\right)^2\right] \tag{9}$$

By inspection it is seen that the maximum velocity occurs at the free surface. Applying the condition $y = 0$ to Equation (9) results in

$$v_{s,max} = \frac{g\Delta^2}{2\nu}\left[1 + \frac{2\mu}{\beta\Delta}\right] \tag{10}$$

To determine the average velocity, the velocity profile is integrated over the film thickness, giving

$$\bar{v}_s = \frac{1}{\Delta}\int_0^\Delta v_s \, dy = \frac{g\Delta^2}{3\nu}\left[1 + \frac{3\mu}{\beta\Delta}\right] \tag{11}$$

The average velocity is taken as the reference velocity and suggests the use of $3\mu/\beta\Delta$ as a correlating parameter. In terms of this parameter and the average velocity, the wall velocity takes the form

$$v_{s,wall} = \frac{g\rho\Delta}{\beta} = \bar{v}_s\left[\frac{\frac{3\mu}{\beta\Delta}}{1 + \frac{3\mu}{\beta\Delta}}\right] \tag{12}$$

In a like manner the maximum velocity becomes

$$v_{s,max} = \frac{3}{2}\bar{v}_s\left[1 - \frac{1}{3}\left\{\frac{\frac{3\mu}{\beta\Delta}}{1 + \frac{3\mu}{\beta\Delta}}\right\}\right] \tag{13}$$

The mass flow rate, per unit length, is then

$$Q_s = \Delta \bar{v}_s = \frac{g\Delta^3}{3\nu}\left[1 + \frac{3\mu}{\beta\Delta}\right] \tag{14}$$

This theoretical development parallels the development in Appendix II. The relations between variables are similar to those equations pertaining to the Nusselt derivation. These relations contain the Nusselt case as a limiting condition. For a finite film thickness Δ, the condition $v_s = 0$ implies $\beta = \infty$. If the condition $\beta = \infty$ is placed in the previous equations, they reduce to the Nusselt relations with the substitution of δ for Δ.

Certain conclusions may be drawn from the preceeding derivations. The mass flow rate in Nusselt flow is

$$Q = \frac{g\delta^3}{3\nu} \tag{15}$$

which on solving for the film thickness results in

$$\delta = \sqrt[3]{\frac{3\nu Q}{g}} \tag{16}$$

The mass flow rate for flow with slip is

$$Q_s = \frac{g\Delta^3}{3\nu}\left[1 + \frac{3\mu}{\beta\Delta}\right] \tag{17}$$

which on solving for the film thickness results in

$$\Delta = \sqrt[3]{\frac{3\nu Q_s}{g}\left[\frac{1}{1 + \frac{3\mu}{\beta\Delta}}\right]} = \sqrt[3]{\frac{0.75(\nu)^2 NRE}{g}\left[\frac{1}{1 + \frac{3\mu}{\beta\Delta}}\right]} \tag{18}$$

If in the slip case and in the Nusselt case the mass flow rate, the liquid and the liquid temperature were the same, the ratio of the thickness with slip to the thickness without slip would be

$$\frac{\Delta}{\delta} = \frac{1}{\sqrt[3]{1 + \frac{3\mu}{\beta\Delta}}} \tag{19}$$

This ratio is always less than one for β finite and greater than zero. If β is infinite the ratio is identically one which means Nusselt flow. The relation between the thickness with slip and the thickness without slip for a film of given mass flow rate, liquid and temperature can be expressed as

$$\Delta = F\delta \text{ where } 0 < F \leq 1 \tag{20}$$

the limiting case in which $F = 1$ is the Nusselt or no slip case.

For equal mass flow rates

$$Q = Q_s \tag{21}$$

or

$$\delta\bar{v} = \Delta\bar{v}_s \tag{22}$$

and it follows that

$$\frac{g\delta^3}{3\nu} = \frac{g\Delta^3}{3\nu}\left[1 + \frac{3\mu}{\beta\Delta}\right] \tag{23}$$

which reduces to, on substitution of Equation (20)

$$\frac{\Delta^3}{\delta^3} = \frac{1}{1 + \frac{3\mu}{\beta\Delta}} = F^3 \tag{24}$$

Hence, the correlating parameter can be expressed in terms of F

$$\frac{3\mu}{\beta\Delta} = \frac{1-F^3}{F^3} \tag{25}$$

The average velocity in Nusselt flow is

$$\bar{v} = \frac{g\delta^2}{3\nu} \tag{26}$$

Applying Equations (20), (25) and (26), Equations (11), (12) and (13) rewritten in terms of F and $\bar{v}$ are

$$\bar{v}_s = \bar{v}\left(\frac{1}{F}\right) \tag{27}$$

$$v_{s,wall} = \bar{v}\left(\frac{1-F^3}{F}\right) \tag{28}$$

and

$$v_{s,max} = \frac{3}{2}\,\bar{v}\left(\frac{2+F^3}{3F}\right) = v_{max}\left(\frac{2+F^3}{3F}\right) \tag{29}$$

From Equations (27) and (29)

$$v_{s,max}\sqrt{v_s} = \frac{2+F^3}{2} \tag{30}$$

The above expressions relate the velocity under conditions of slip flow to velocity at the same mass flow rate if there were no slip at the wall.

In order to calculate slip velocities, it is first necessary to calculate the average velocity that a given liquid would have for a certain mass flow rate and temperature if it was flowing under Nusselt conditions. In Table10 calculated values for the average velocity, film thickness and mass flow rate of Nusselt flow, as a function of Reynolds number, are given. These values are calculated from Equations (47), (57) and (56) respectively as they are presented in Appendix II. The values of the Reynolds number selected cover the range of experimental

Table 10. Average Film Velocity, Film Thickness and Mass Flow Rate as a Function of Reynolds Number for Theoretical Nusselt Film Flow Conditions

Reynolds Number dimensionless	Mass Flow Rate $cm.^2/sec.$	Film Thickness cm. x 100	Average Velocity cm./sec.
1	0.0022	0.395	0.57
10	0.0224	0.851	2.63
50	0.1120	1.455	7.70
100	0.2241	1.833	12.23
200	0.4482	2.310	19.40
500	1.1204	3.135	35.74
1000	2.2409	3.950	56.73

values of this work.

Once the Nusselt velocity has been obtained from Table 10, the slip velocities are obtained by multiplying the Nusselt velocity by the average velocity factor, or the wall velocity factor, from Table 11. The maximum velocity factor given in the last column shows the relative increase in the maximum or surface velocity as a function of the amount of film reduction due to slip. Table 11 summarizes the F functions in terms of F, the relative film reduction due to slip at the wall. In other words, the functions of F are given in terms of F.

As an example, assume that water is flowing down a vertical plate with a Reynolds number of 200 at 25° C. From Table 10, the theoretical Nusselt thickness is 2.310×10^{-2} cm. and the average velocity is 19.40 cm./sec. If instead of Nusselt flow the film is slipping at the wall to such an extent that there is a five percent reduction in film thickness, the value of F is 0.95. From Table 11, the average velocity factor, $\frac{1}{F}$, is 1.053 so the average velocity with slip would be $1.053 \times 19.40 = 20.43$ cm./sec. In a similar manner, the wall velocity factor, $\frac{1-F^3}{F}$, equals 0.150 so the wall velocity would be $0.150 \times 19.40 = 2.91$ cm./sec. Finally, the maximum, or surface, velocity would be $1.003 \times 19.40 = 19.46$ cm./sec.

Analysis of the Theoretical Derivation

The effect of slip at the wall on two properties of thin films, thickness and velocity, was studied. Pertinent observations and the results of this study follow.

Table 11. Velocity Factors as a Function of Relative Film Thickness

Relative Film Thickness	Velocity Factor		
	Average	Wall	Maximum
F	$\frac{\bar{v}_s}{\bar{v}} = \frac{1}{F}$	$\frac{v_{s,wall}}{\bar{v}} = \frac{1-F^3}{F}$	$\frac{v_{s,max}}{v_{max}} = \frac{2+F^3}{3F}$
1.000	1.000	0.000	1.000
0.999	1.001	0.003	1.000
0.990	1.010	0.030	1.000
0.980	1.020	0.060	1.000
0.950	1.053	0.150	1.003
0.900	1.111	0.301	1.011
0.850	1.176	0.454	1.025
0.800	1.250	0.610	1.047
0.700	1.429	0.939	1.116

Film Thickness

If slip is present, one indication would be a thinner film. This is shown by Equation (19). In each of the four series which were run with water and the low-middle flow rate series which was run with the glycerine solution, the minimum average film thickness was found to be less than the Nusselt thickness.

If slip occurs when a liquid flows over a solid surface, then the liquid cannot be completely wetting the surface because the surface layer of the liquid is moving with respect to the solid-liquid boundary. An increase in the wettability of the liquid should result in an increase in the film thickness. This was found to be the case when Aerosol-OT was added to the water in an amount sufficient to cause a 50 percent reduction in the surface tension. Figures 14 and 15 show that the films were thicker with the aerosol-OT solution than with pure water at equivalent flow rates. Two different flow rates were used in these experiments and in each case the addition of the Aerosol-OT caused a thicker film.

It was demonstrated that an increase in the wettability of a given surface by a liquid caused an increase in the thickness of a liquid film flowing over the surface. Following the same argument, a decrease in the wettability should result in a decrease in film thickness. To check this experimentally, the flat vertical plate was coated with a thin layer of paraffin and an attempt was made to repeat some of the water tests. However, under these conditions, it was found to be impossible for the water to flow down the plate in the form of a thin film. Immediately after the water started down the plate the edges of the film began to move toward the center, and within five or six inches

from the top of the plate all of the water was flowing down the middle in a relatively thick stream about an inch wide.

An Aerosol-OT solution should wet a paraffin surface better than pure water, and it was found that a flowing film of this solution could be maintained on the paraffin-coated vertical plate. Since the degree of wetting of the Aerosol-OT solution on the paraffin surface is much less than on the aluminum surface, a film should slip more on the paraffin surface at the equivalent flow rate with a resultant decrease in film thickness. This was verified experimentally, and a few tests were made using the Aerosol-OT solution and the paraffin-coated plate. The results of the experiment are shown in Figure 17 from which it may be seen that the paraffin coating caused thinner films.

Film Velocities

The measurement of point velocities, or of the velocity profile, in vertically falling liquid films is most difficult because the film is so thin. To date no satisfactory method has been developed and very few reports of film profiles have been published. However, even if such a method existed, it would afford a poor means for determining whether or not slip were present. The tabulation of the calculated effect of slip on the maximum velocity, shown in Table 11, indicates that if slip were present to the extent that a reduction in film thickness of ten percent occurred, it would only result in a one percent increase in the maximum velocity of the film.

The liquid velocities down the wall calculated in this section do not appear unreasonable for modest reductions in film thickness. In any case, no experimental evidence has been reported on measured liquid

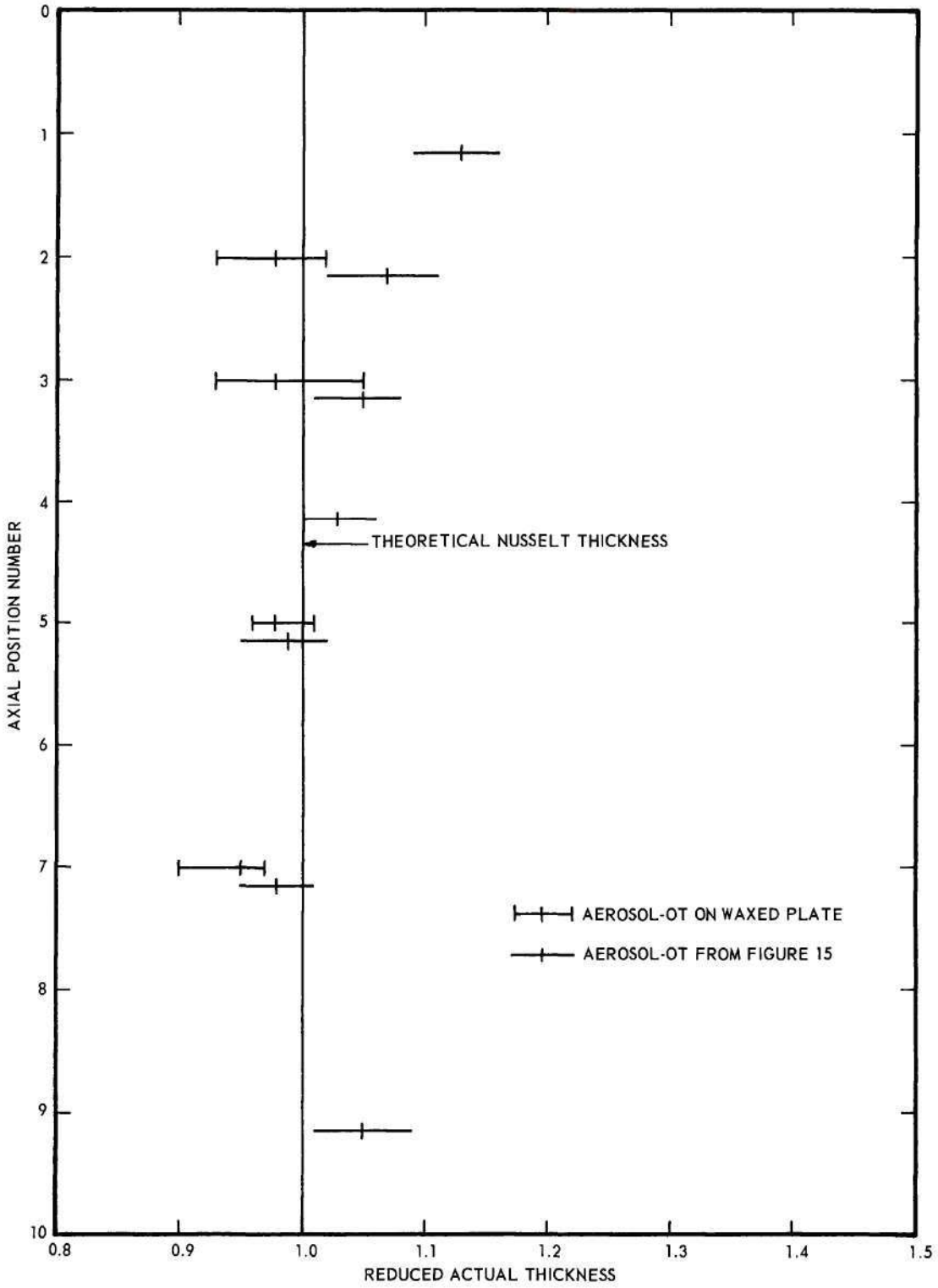

Figure 17. Mean Film Thickness and Wave Amplitude for the Aerosol-OT Solution on a Waxed Flat Plate.

velocity at the wall, perhaps due to the assumption made that they are zero.

The major portion of the published work which is concerned with the velocities of thin falling films deals with the ratio of the maximum velocity (the velocity at the air-liquid interface) to the average velocity. This ratio was determined experimentally by measuring the average film thickness, the mass flow rate and the surface velocity. Equation (54) in Appendix II shows that this ratio is 1.5 for Nusselt flow. From Equation (30) this ratio is equal to $\frac{2+F^3}{2}$ and is always less than 1.5 for F less than one, that is, for conditions of slip flow. Values of this velocity-ratio factor for various conditions of slip flow are given in Table 12.

The first workers who reported experimental values of this ratio include Friedman and Miller,[31] Grimley [9] and Chew[38] who timed the movement of dye drops on the free surface to obtain the maximum velocity. Brauer[15] and Jaymond[39] used plastic particles as surface tracers; Asbjornsen[40] employed a residence time technique; and Jackson, Johnson and Ceaglske[41] made use of a pressure drop measurement technique to give the surface velocity of the thin liquid films. In general, the results of these investigators agree that, for extremely low flow rates where there is no noticeable wavy surface, the value of the velocity ratio is about 1.5. For higher flow rates where there were well-defined ripples, the velocity ratio increased to about two which was followed by a gradual reduction back to 1.5 as turbulent film flow was approached.

More recent results have not agreed with these data of the early investigators. Fulford[6] calculated surface velocities from the wave-

Table 12. Maximum Velocity-Average Velocity Ratio for Slip Flow Conditions

Film Reduction Factor F	Ratio Reduction Factor $\frac{2+F^3}{2}$
1.000	1.500
0.999	1.499
0.990	1.485
0.980	1.471
0.950	1.429
0.900	1.364
0.850	1.307
0.800	1.256
0.700	1.172

lengths of standing waves formed on the film surface upstream of a stationary pointer just touching the liquid surface. Experimental ratios, indirectly measured, were about equal to the theoretical Nusselt value of 1.5 with no tendency to increase as previously reported. A rather wide scatter of experimental data (in the author's words) averaged 1.543.

A paper by Portalski[42] in 1964 sheds an interesting light on this problem with regard to the slip hypothesis. Although the author states the velocity ratios fell close to 1.5 and used this as an argument in favor of parabolic film flow, with no slip at the wall, an examination of the reported data proves most interesting. Seven different cases are shown; water on a mild steel plate; water on a stainless steel plate; water with two different surface-active agents; and three different concentrations of glycerine in water, 45, 71.5 and 75 percent glycerine by weight. In all of these the only ones which exhibited values for the velocity ratio over 1.5 were the two highest-concentration glycerine solutions and the two solutions with surface-active additions. In the 71.5 percent glycerine solution, four out of 14 reported ratios were 1.5 or greater. The average was 1.47 and the range was 1.37 to 1.56. The 75 percent glycerine solution had only one out of ten points 1.5 or greater. For both water series and the 45 percent glycerine series all the ratios were below 1.5.

The results reported in the surface-active agent series showed an unusual trend. At the lower flow rates the velocity ratio was less than 1.5; at the higher laminar flow rates the ratio was greater than 1.5; and at still higher flow rates, near where the film flow became

turbulent, the ratio dropped below 1.5.

Portalski's data, discounting a few individual points in the high concentration glycerine series, show that the velocity ratio for pure water and water-glycerine solutions is less than 1.5. Portalski's data also show that the effect of the addition of a surface-active agent is to increase this velocity ratio. This is in agreement with the results of this investigation and implies slip at the solid-liquid interface.

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

The following conclusions have been drawn from the results of this study.

1. A different and unique system has been developed for the measurement of liquid film thicknesses which allows the observation of the position of the dynamic film surface without the introduction of foreign materials or objects into the liquid surface.

2. An average film thickness based on the entire flow area or on one point in the flow area is inadequate for describing film thickness as the film thickness of a liquid in vertical flow at constant flow rate is a function of the position on the plate at which the measurement is made.

3. The wave amplitudes of the surface disturbances on vertical film flow at constant flow rate are dependent on the position on the surface at which the measurement is made.

4. An increase in the viscosity of water by the addition of glycerine increases the film thickness and wave amplitude but does not alter the general film-thickness profile.

5. A decrease in the surface tension of water by the addition of a surface-active material increases the film thickness and decreases the wave amplitude at constant flow rate. In comparison with a pure water flow, water containing a surface-active material has a relatively

constant film thickness profile over the entire plate and the film surface is comparatively smooth.

6. A new and different method for determining the distance before wave inception has been developed which is more sensitive than previous visual or photographic methods.

7. In the smooth flow region before wave inception, all liquids investigated exhibited average film thicknesses less than the theoretical Nusselt thickness except the glycerine solution at the high flow rate.

8. Slip at the solid-liquid interface is a possible explanation for the occurrence of films thinner than the Nusselt prediction.

9. The observed increase in film thickness on the addition of a surface active material to water and the decrease in film thickness when a liquid is passed over a waxed plate could result from slip at the solid-liquid interface.

The results of this study have indicated several further investigations which would be fruitful.

1. The range of flow rates studied could be extended to lower Reynolds numbers which would settle the question of whether or not the distance before wave inception increases or decreases as the flow is reduced below a Reynolds number of 75.

2. At the lower flow rates the question of whether the film thickness approaches the Nusselt theory could be determined.

3. Perhaps the most interesting point to investigate would be the apparent indication of slip at the solid-liquid interface which should be more noticeable at the lower flow rates.

4. The effect of the entrance velocity profile on the film thickness profile could be investigated for several configurations of the entrance distributor for the liquid.

5. The measuring equipment developed in this work could be refined and adapted to other flow conditions. One possible refinement would be an increase in the speed at which measurements were made.

APPENDICES

APPENDIX I

CONSTRUCTION DETAILS

The following sections contain detailed information pertaining to the various components which composed the thickness measuring system.

Flow Plate

An aluminum plate (Kaiser Alclad) four feet long, two feet wide and 1/8 inch thick was used to provide the surface over which the liquids flowed in this investigation. Prior to use, the surface of the plate was treated with concentrated hydrochloric acid, washed with a strong detergent solution and thoroughly rinsed with water. The plate was fastened with countersunk machine bolts to a 3/4 inch plywood backing sheet of the same length and width. The plate and backing sheet were mounted on two 1-1/2 by 2-1/2 inch angle irons four feet long which had been pre-drilled along the sides by the manufacturer. A calming header, which contained four baffles, was attached to the upper part of the plate assembly. These baffles were sufficient to insure a smooth and steady flow of liquid onto the flow section of the plate. The flow section was the center 12 inches of the plate. Hence, the wetted flow area was one foot wide and four feet long. This entire assembly, mounted on the two steel angle irons, was movable up and down the main equipment rack which was also constructed out of the pre-drilled angle iron.

Flow System

A constant head tank 1-1/2 feet long, one foot wide and three feet deep was used to supply the liquid being tested to the calming header. This tank was provided with two overflow lines, one of which was on the opposite side of the tank and slightly above the other. The overflow for the tank as well as the liquid which passed over the plate were collected into a lower collection tank, two feet wide, four feet long and 1-1/2 feet deep.

For the water series the constant head tank was supplied from the city mains, and the water after reaching the lower collection tank was discarded. However, for the other two series, the solutions were not discarded until all tests with that solution were completed, and during the runs the constant head tank was supplied by recirculation from the lower collection tank. A Berkeley M 1030 pump driven by a 1/4 horsepower induction motor was used for this purpose.

Measuring System

The light source used in this work was a 1000 watt projector bulb mounted in a modified projector housing. The light beam was collimated by two collimators prior to reaching the flow plate.

The lens, which was focused on the centerline of the wetted flow area, was a Bausch and Lomb aerial camera lens with a focal length of 345 mm. This lens transferred the object image, located on this centerline, to the film holder (camera) which was about 15 feet from the lens.

The camera used throughout this experimental investigation was

a Miranda Model F, single lens reflex, with a f1.9 lens. The lens was removed in all film thickness and calibration runs except for the identification exposures made at the beginning and again at the end of each run. The film used was Kodak Tri-X Pan 35 mm. film.

Film Processing

The development procedure for the exposed film was as follows:

Kodak D-19 Developer (full strength)	10 minutes
Kodak Indicating Stop Bath	30 seconds
Kodak Acid Fixer	10 minutes
Water Wash (running water)	30 minutes
Kodak Photoflo	30 seconds
Air Drying	As required

APPENDIX II

NUSSELT FILM THEORY

The flow of viscous liquids in thin films is complicated by the appearance on the film surface of waves and ripples. The surface phenomena greatly complicate any mathematical model used to characterize the flow of thin films. However, if certain simplifying assumptions are made, relations may be developed which will give an insight into the film behavior under limiting conditions. The following development as originally derived by Hopf[43] and Nusselt[3] outlines the simplest case.

The most general equations for laminar flow of a viscous incompressible fluid of constant physical properties are the Navier-Stokes equations. Using the standard rectangular coordinate system in terms of x, y, and z, the Navier-Stokes equations may be written:

$$v \frac{\partial v}{\partial x} + u \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} = - \frac{\partial \Omega}{\partial x} - \frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \nabla^2 v \qquad (31)$$

$$v \frac{\partial u}{\partial x} + u \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} = - \frac{\partial \Omega}{\partial y} - \frac{1}{\rho} \frac{\partial P}{\partial y} + \nu \Delta^2 u \qquad (32)$$

and

$$v \frac{\partial w}{\partial x} + u \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t} = - \frac{\partial \Omega}{\partial z} - \frac{1}{\rho} \frac{\partial P}{\partial z} + \nu \nabla^2 w \qquad (33)$$

where v, u, w are the fluid velocities in the x, y, z directions, t is the time, ρ and ν are the density and the kinematic viscosity, P is the pressure, Ω is the force potential of the field in which the fluid flows, and ∇^2 is the Laplacian operator.

With this convention the continuity equation is

$$\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{34}$$

Since the only force to be considered in the vertical flow of fluids is gravity, the negative derivitives of Ω are equal to gravity in the x-direction and zero elsewhere. The following coordinate convention will be used throughout the discussion. The x-axis is in the major direction of flow, that is, the vertical direction. The y-axis is perpendicular to the plate and the z-axis is across the plate, both are horizontal. The coordinate system and flow variables are shown in Figure 16 in Chapter III.

If the flow is steady, uniform and two-dimensional, the Navier-Stokes equations reduce to the following while the continuity equation is automatically satisfied.

$$\frac{d^2v}{dy^2} + \frac{g}{\nu} = 0 \tag{35}$$

$$\frac{dP}{dy} = 0 \tag{36}$$

and

$$\frac{dP}{dz} = 0 \tag{37}$$

Integrating Equation (35) gives

$$\frac{dv}{dy} = -\frac{g}{\nu} y + C_1 \tag{38}$$

where C_1 is the constant of integration.

Application of the boundary condition of no shear at the liquid-gas interface, which is the origin of the coordinate system, means that

$$\frac{dv}{dy} = 0 \qquad \text{at } y = 0 \tag{39}$$

and results in

$$C_1 = 0 \tag{40}$$

Equation (38) reduces to

$$\frac{dv}{dy} = - \frac{g}{\nu} y \tag{41}$$

Integrating Equation (41) gives

$$v = - \frac{gy^2}{2\nu} + C_2 \tag{42}$$

where C_2 is the second constant of integration.

Applying the second boundary condition of no slip at the wall, where δ is the liquid film thickness, or

$$v = 0 \qquad \text{at } y = \delta \tag{43}$$

results in the solution

$$C_2 = \frac{g\delta^2}{2\nu} \tag{44}$$

Substituting the value of C_2 from Equation (44) into Equation (42) and rearranging results in the velocity profile for the liquid flow and gives

$$v = \frac{g\delta^2}{2\nu} \left[1 - \left(\frac{y}{\delta}\right)^2\right] \tag{45}$$

By integrating the velocity profile, Equation (45), over the film thickness δ, the mean velocity is found to be

$$\bar{v} = \frac{1}{\delta} \int_0^{\delta} v \, dy = \frac{g\delta^2}{3\nu} \tag{46}$$

The volumetric flowrate per unit wetted-perimeter is then

$$Q = \delta\bar{v} = \frac{g\delta^3}{3\nu} \tag{47}$$

The following quantities may be obtained from the above relations:

$$\bar{v} = \frac{g\delta^2}{3\nu} \tag{48}$$

$$v = \frac{g\delta^2}{2\nu}\left[1 - \left(\frac{y}{\delta}\right)^2\right] = \frac{3}{2}\frac{g\delta^2}{3\nu}\left[1 - \left(\frac{y}{\delta}\right)^2\right] = \frac{3}{2}\bar{v}\left[1 - \left(\frac{y}{\delta}\right)^2\right] \tag{49}$$

$$v_{wall} = \frac{3}{2}\bar{v}\left[1 - \left(\frac{y}{\delta}\right)^2\right]_{y=\delta} = 0 \tag{50}$$

$$v_{max} = \frac{3}{2}\bar{v}\left[1 - \left(\frac{y}{\delta}\right)^2\right]_{y=0} = \frac{3}{2}\bar{v} \tag{51}$$

$$v_{max}/\bar{v} = \frac{3}{2}\bar{v}/\bar{v} = \frac{3}{2} \tag{52}$$

$$Q = \frac{g\delta^3}{3\nu} = \delta\bar{v} \tag{53}$$

Solving Equation (53) for the film thickness, δ, results in

$$\delta = \text{Theoretical Nusselt Thickness} = \text{TNT} = \sqrt[3]{\frac{3\nu Q}{g}} \tag{54}$$

The reduced actual thickness, RAT, is defined by

$$\text{Reduced Actual Thickness} = \text{RAT} = \frac{\text{Actual Thickness}}{\text{Theoretical Nusselt Thickness}}$$

$$= \frac{\text{AT}}{\text{TNT}} \tag{55}$$

The Reynolds number as used throughout this discussion, NRE, is

$$\text{Reynolds Number} = \text{NRE} = \frac{4\,Q}{\nu} \tag{56}$$

Solving Equation (56) for Q and substituting this into Equation (54)

results in

$$TNT = \sqrt[3]{\frac{0.75\ (v)^2\ NRE}{g}} \tag{57}$$

APPENDIX III

TABULAR FILM THICKNESS DATA

Table 13. Film Thickness for Water at the Low Flow Rate

Position Number	Reduced Actual Thickness			Theoretical Nusselt Thickness (cm. x 10^2)
	Minimum	Average	Maximum	
0	0.48	0.50	0.59	1.88
1	0.69	0.75	0.83	1.95
2	0.61	0.73	0.83	1.93
3	0.45	0.66	0.81	1.97
4	0.53	0.85	1.03	1.97
5	0.69	1.02	1.18	1.99
5	0.71	1.06	1.27	2.01
6	0.82	1.09	1.44	2.04

Table 14. Film Thickness for Water at the Low-Middle Flow Rate

Position Number	Reduced Actual Thickness			Theoretical Nusselt Thickness (cm. x 10^2)
	Minimum	Average	Maximum	
0	1.00	1.07	1.15	2.79
1	0.84	0.89	0.94	2.79
2	0.83	0.88	0.96	2.80
3	0.84	1.13	1.36	2.86
3	0.80	1.23	1.62	2.82
4	0.70	1.13	1.40	2.80
5	0.91	1.25	1.66	2.86
6	0.82	1.34	1.74	2.83

Table 15. Film Thickness for Water at the High-Middle Flow Rate

Position Number	Reduced Actual Thickness			Theoretical Nusselt Thickness (cm. x 10^2)
	Minimum	Average	Maximum	
0	1.63	1.71	1.78	3.58
1	0.99	1.02	1.07	3.62
2	0.93	0.95	0.99	3.63
3	0.89	0.94	1.01	3.64
4	0.79	0.88	1.07	3.65
5	0.88	1.05	1.28	3.70
6	0.72	1.06	1.38	3.66
7	0.94	1.25	1.74	3.65
7	0.79	1.36	2.00	3.63
8	0.89	1.53	2.30	3.63
8	1.03	1.61	2.14	3.60

Table 16. Film Thickness for Water at the High Flow Rate

Position Number	Reduced Actual Thickness			Theoretical Nusselt Thickness (cm. x 10^2)
	Minimum	Average	Maximum	
0	1.96	2.18	2.35	4.02
1	1.07	1.09	1.15	4.11
2	0.91	0.95	0.98	4.11
3	0.97	1.01	1.04	4.16
3	0.96	0.99	1.04	4.17
4	0.87	0.90	0.94	4.16
5	0.90	0.97	1.03	4.17
6	0.94	0.98	1.04	4.20
7	0.87	1.02	1.21	4.14
7	0.88	1.21	1.64	4.15
7	0.85	1.31	1.81	4.17
8	0.91	1.26	1.74	4.16
8	0.81	1.30	1.83	4.10
8	0.89	1.41	2.04	4.13

Table 17. Film Thickness for the Glycerine Solution at the Low-Middle Flow Rate

Position Number	Reduced Actual Thickness			Theoretical Nusselt Thickness (cm. x 10^2)
	Minimum	Average	Maximum	
0	1.16	1.22	1.25	3.87
1	0.92	0.94	0.96	3.90
2	0.94	1.01	1.09	3.89
3	0.91	1.04	1.39	3.94
4	0.76	1.13	1.75	3.97
5	0.77	1.31	1.73	3.93

Table 18. Film Thickness for the Glycerine Solution at the High Flow Rate

Position Number	Reduced Actual Thickness			Theoretical Nusselt Thickness (cm. x 10^2)
	Minimum	Average	Maximum	
0	2.59	2.75	2.92	5.84
1	1.22	1.41	1.58	5.77
2	1.09	1.15	1.20	5.79
3	1.03	1.16	1.29	5.86
4	1.07	1.11	1.14	5.82
5	0.90	1.02	1.08	5.88
6	1.08	1.13	1.20	5.80
7	0.89	1.12	1.60	5.81
8	1.11	1.25	1.81	5.82

Table 19. Film Thickness for the Aerosol Solution at the Low Flow Rate

Position Number	Reduced Actual Thickness			Theoretical Nusselt Thickness (cm. x 10^2)
	Minimum	Average	Maximum	
0	0.62	0.64	0.66	1.90
1	0.83	0.90	0.98	1.89
2	0.99	1.03	1.06	1.96
3	0.86	0.90	0.96	1.96
4	0.96	1.02	1.07	1.96
5	0.90	0.96	1.01	1.97
7	0.84	0.87	0.90	2.06
9	0.95	0.98	1.02	2.01

Table 20. Film Thickness for the Aerosol Solution at the High-Middle Flow Rate

Position Number	Reduced Actual Thickness			Theoretical Nusselt Thickness ($cm. \times 10^2$)
	Minimum	Average	Maximum	
0	1.56	1.67	1.78	3.24
1	1.09	1.13	1.16	3.25
2	1.02	1.07	1.11	3.23
3	1.01	1.05	1.08	3.23
4	1.01	1.03	1.06	3.25
5	0.95	0.99	1.02	3.26
7	0.95	0.98	1.01	3.36
9	1.01	1.05	1.09	3.32

Table 21. Film Thickness for the Aerosol Solution on a Flat Waxed Plate

Position Number	Reduced Actual Thickness			Theoretical Nusselt Thickness (cm. x 10^2)
	Minimum	Average	Maximum	
2	0.93	0.98	1.02	3.18
3	0.93	0.98	1.05	3.16
5	0.96	0.98	1.01	3.11
7	0.90	0.95	0.97	3.07

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